Magnetically assisted gas–solid fluidization in a tapered vessel: Part I. Magnetization-LAST mode

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Abstract

This article presents further experimental results of the Magnetization-LAST mode in magnetically assisted gas-fluidized tapered beds, including external transverse magnetic field control of solid phase movement, central channel formation, spout depth and the pressure drop across the bed. Phase diagrams similar to those recently reported for the Magnetization-FIRST mode were also developed. Dimensional analysis based on “pressure transform” of the initial set of variables and involving the magnetic granular Bond number pertinent to particle aggregate formation was applied to develop the scaling relationships.

Keywords: Fluidization; Magnetization-LAST; Tapered bed; Dimensional analysis; Pressure transform of variables; Magnetic granular Bond number

1. Introduction

Originally conceived by Mathur and Epstein (1974) for gas-fluidization of Geldart D particles (Geldart, 1973), the use of tapered vessels has been extended to the fluidization of cohesive particles (Venkatesh, Chaouki, & Klína, 1996; Erbil, 1998), e.g., the drying of agriculture by-products (Wachiraphansakul & Devahastin, 2007) and pasty materials (Bacelos, Passos, & Freire, 2007; Bacelos & Freire, 2008). Such sticky solids with high liquid content and interparticle capillary forces cause easy particle aggregation and affect adversely the fluid-bed hydrodynamics (Bacelos et al., 2007; Bacelos & Freire, 2008).

In magnetically assisted fluidization a magnetic field controls, remotely and outside the fluidized bed, interparticle forces between solid particles that are magnetic while fluidized by either a liquid (Hristov, 2006), a gas (Hristov, 2002, 2003a) or gas–liquid flow (Hristov, 2008), in either of two basic magnetization modes, FIRST (magnetization of a fixed bed before fluidization) and LAST (magnetization of a preliminarily fluidized bed). The hydrodynamic behaviour of magnetic fluidization of coarse Geldart B particles was reviewed by Hristov (2002, 2003a, 2003b, 2004, 2006, 2007a). Recently, promising results on magnetically assisted nano-fluidization were reported by Hao et al. (Hao, Zhu, Lei, & Li, 2008a; Hao, Zhu, Jiang, & Li, 2008b) on mass transfer pertinent to CH4–CO2 reforming.

The present article addresses magnetically assisted spouted beds with the Magnetization-LAST mode as a continuation of the recent report (Hristov, 2008) on the Magnetization-FIRST mode. For such Magnetically Assisted Tapered Bed (MATB) we shall demonstrate how the external magnetic field controls the bed behaviour through induced interparticle forces and aggregate formation. The microscopic phenomena at the particle diameter scale finally affect the gross bed behaviour in terms of pressure drop, bed depth, etc.

In this work, we used a single tapered vessel with a fixed cone angle in the Magnetization-LAST mode (Hristov, 1998, 1999, 2002). The magnetic field was generated by saddle coils oriented normally to the fluid flow and the vessel axis as detailed by Hristov (2008). The following basic features pertinent to the effects of magnetic field are to be addressed to:

- Bed magnetic behaviour, pressure drop histories, bed collapse (porosity) variations and the related physics due to variation in the intensity of the external magnetic field.
- Critical field intensities and the shifts of regimes as affected by solid mass charged to the vessel, particle size, etc.
Nomenclature

$A_0, A_{p0}$ dimensionless coefficients defined by Eq. (7a)

$A_p$ dimensionless coefficient defined by Eq. (4d)

$B_{g-c} = P_c/P_g = P_c/\rho_s g d_p$ Bond number of granular materials with a natural cohesion

$B_{g-m} = (\mu_0 M_s H)/(\rho_s g d_p)$ magnetic Bond number for granular materials

$c_0$ dimensionless coefficient in Eq. (7a)

$C_D = \Delta P_0/2\rho_f U_f^2$ tapered bed friction factor before the magnetic field application (–)

$D_b$ diameter of the flow entrance (m)

$D_L$ tube diameter (diameter of the cylindrical section above the cone) (m)

$D_{bL}$ dimensionless ratio of bed geometric characteristics defined by Eq. (2c) (–)

$d_p$ particle diameter (m)

$E_1, E_2$ dimensionless functions defined by Eqs. (11a) and (11b) (–)

$e_1, e_2, e_3, e_4$ dimensionless exponents in Eqs. (11a) and (11b) (–)

$f_1, f_2$ dimensionless functions defined by Eqs. (4a) and (4b)

$f_{10} = G a \Delta D_{bL}$ dimensionless function (Eqs. (7a) and (7b))

$f_n, f_{n1}, f_{n2}$ dimensionless functions defined by Eqs. (10a) and (10b)

$G$ mass of particle bed (kg)

$G_a = (d_p^2 g \rho_f^2)/\eta_f^2$ Galileo number

$g$ gravity acceleration, 9.81 m/s$^2$

$H$ magnetic field intensity (A/m)

$H_{st}$ magnetic field intensity at which the particle aggregation begins (A/m)

$H_{f-1}$ minimum freezing magnetic field intensity at onset of top frozen bed (A/m)

$H_{f-2}$ minimum freezing magnetic field intensity required for complete bed freezing (A/m)

$h_b$ bed height (m)

$h_{b0}$ initial bed height (m)

$h_L$ bed length at the wall (see Fig. 1) (m)

$h_{ID}$ initial bed length at the wall (see Fig. 1) (m)

$h_U$ bed depth before the magnetic field application (see Eq. (10a)) (m)

$M_s$ magnetization at saturation (A/m)

$M_p$ mass of particles charged into the vessel (kg)

$m$ dimensionless exponent in Eq. (10b) (–)

$n_p, n_{p1}, n_{p2}$ dimensionless exponents (–)

$P_c$ cohesion (Pa)

$P_g = \rho_s g d_p$ gravity pressure peer unit surface of interparticle contacts (Pa)

$\Delta P$ pressure drop (Pa)

$\Delta P_0$ pressure drop across the preliminary fluidized bed prior to magnetization (Pa)

$Q$ volumetric gas flow rate (m$^3$/s)

$Q_{0f0}$ volumetric gas flow rate at the minimum fluidization point in absence of a field (m$^3$/s)

$Q_{ms}$ volumetric gas flow rate at the minimum spouting point in absence of a field (m$^3$/s)

$U$ superficial gas velocity (denoted also as $U_f$ or $U_0$—see the text) (m/s)

$V_p = M_p/\rho_s$ volume occupied by the solids (m$^3$)

$V_{bed} = \pi h_{ID}(D_L^2 + D_L D_b + D_b^2)$ (m$^3$)

Greek letters

$\alpha$ cone angle (°)

$\varepsilon$ porosity (–)

$\varepsilon_0$ initial bed porosity (–)

$\varepsilon_a$ annular bed porosity (–)

$\mu$ magnetic permeability (Wb/A m) or (H/m)

$\mu_0$ magnetic permeability of the space (Wb/A m)

$\eta_f$ fluid dynamic viscosity (denoted also as $\eta$ for simplicity of the expressions) (Pa s)

$\rho_f$ fluid density (kg/m$^3$)

$\rho_g$ gas density (kg/m$^3$)

$\rho_s$ solid particle density (kg/m$^3$)

Subscripts

$f$ fluid

$p$ particle

$s$ solids

- Dimensional analysis of both the effect of the conical shape of vessel and the presence of the magnetic field, including data correlation.

2. Experimental

The experimental set-up, described in detail elsewhere (Hristov, 2008), consists of a conical vessel (15° opening angle, 30-mm-ID bottom diameter and 190-mm-ID top diameter)
surrounded by 200-mm-ID and 400-mm-high saddle coils (Hristov, 2002, 2008). The field lines were oriented transversely to the cone axis and fluid flow, as shown in Fig. 1. The magnetic field was steady with maximum intensity of about 40 kA/m. Magnetite sand (315–400 \( \mu \)m, \( \rho_s = 5100 \text{ kg/m}^3 \), \( M_s = 477.37 \text{ kA/m} \)) was fluidized by air in the experiments. A mechanical valve and a calibrated rotameter were used to control and measure the gas flow rate. Pressure drop was measured by an U-tube water manometer connected between the gas inlet and a fine-tube pressure probe placed above the bed top surface.

3. Results

3.1. Phase diagrams

Fig. 2 describes the flow regimes in a typical phase diagram in \( Q-H \) coordinates (Hristov, 2008), showing that with increasing intensity \( H \) the originally fluidized bed passes through three basic states as described in Fig. 3: (a) a fluidized bed with minor effects of the magnetic field on smooth fluidization with wave-
like slits up to violent bubbling or spouting; (b) fluidization topped by a frozen top layer which grows progressively in depth with increasing magnetic field intensity; (c) a frozen bed entirely immobilized due to magnetic coagulation. At low gas flow rates the initial bed is fluidized \((Q > Q_{mf})\) with bubbles and slits to which the field is applied (Fig. 3B). The freezing is progressive, starts at the intensity \(H_{fr-1}\) and the frozen layer \((FrB)\) propagates toward the bed bottom. The final frozen bed at \(H > H_{fr-2}\) is not homogeneous and may contain cavities (voids) as “frozen bubbles”) or to be arranged as a layered structure of particle layers separated by slits. In all these stages the pressure drop decreases as the field intensity is increased.

At high flow rates enough to create spouted bed \((Q > Q_{ms})\) a stable central fountains is formed and field applied tries to suppress the spouting. The bed top becomes frozen at the top immediately after reaching \(H > H_{fr-1}\) without visual detectable strings. The frozen bed section \((FrB)\) propagate downward and in some cases, especially at high gas flow rates, a minor section in the neighbourhood of the gas inlet remains fluidized though it does not extends upward to represent the gross bed behaviour. The final frozen bed \(H > H_{fr-2}\) commonly contains an extended gas void as a remnant of the central spout channel.

Detectable field effects occur beyond the boundary defined by the freezing of the top bed layer. In this frozen layer the superficial gas velocity is the lowest, and with increasing magnetic field intensity particle aggregation takes place first. It resembles the top layer of MSB (magnetically stabilized bed) in a cylindrical vessel for the Magnetization-FIRST mode (Hristov, 2008) since in both cases magnetic flocculation dominates and fluid drag is not enough to prevent it. However, the origins and structures of the frozen sections are different. For Magnetization-FIRST the frozen section (MSB) is the remnant of an originally magnetized fixed particle bed, while for the present case the frozen bed is formed by settled particle flocks formed from particles freely suspended in the gas flow. In contrast with cylindrical beds (Hristov, 1998, 1999, 2002) where this freezing occurs simultaneously over the entire bed, the tapered geometry allows this to happen first at the bed top.

The continuously growing frozen section, from the top toward the gas inlet as the magnetic field increases incrementally by small steps is capable of suppressing by degrees particle motion as well as bubbling and internal fountain formation. With the 3D vessels used in the present study it was difficult to measure the geometric characteristics of the bed sub-sections, so only the overall bed depth and its qualitative behaviour were recorded. Certainly, this calls for special experiments on gas spout evolution with both magnetization modes that surely would involve 2D experimental vessels for easier detection of bed internal structure. However, this is beyond the scope of the present report.

### 3.2. Bed collapse and pressure drop

Increase in magnetic field intensity causes particle polarization and augmentation of attractive interparticle forces that finally leads to particle flocculation. That is, increase in size of the fluidized flocks, instead of single particles, decelerates the motion of the solids in the bed as is manifested by bed collapse, i.e. the magnetized flock tries to minimize its volume, as a natural phenomenon analogous to gravity in Newton’s law and described by Earnshaw’s theorem (Hristov, 1998, 2002). The collapsed bed has lower porosity than the initial fluidized bed whose expansion depends on gas flow rate. Higher bed expansion of the initially fluidized bed calls for higher energy introduced by the flowing gas that requires higher magnetic field intensity to collapse (shrink) the fluidized bed. This is clearly demonstrated by the phase diagrams where both curves \(H > H_{fr-1} = f(Q/Q_{ms})\): onset of the freezing bed top layer (FBTL) and that of completer bed freezing \((H_{fr-2})\) go up with increasing gas flow rate.

Fig. 4A shows the evolution of pressure drop across the collapsing bed as a function of increasing magnetic field intensity. In general, the pressure drop decreases, irrespective of oscillations due to formation of internal bed structures (e.g., spouting); eventually approaching some stable value at high field intensities. Both the initial (non-magnetized fluidized bed) and the final (frozen bed) pressure drops depend on gas flow rate established before the application of the magnetic field. As a rule, low gas flow rates give smoother curves since only bubbles as slits or more violent void formations exist in the bed. The formation of a spouted bed before magnetic field application and the persistence of a spout beneath the growing frozen bed at the bed top section cause pressure drop pulsations, as shown by the curves in Fig. 4A and the inset, that does not alter in general the trend in pressure drop with increasing magnetic field intensity.

Collapsing bed changes its overall porosity, and the general trend is decreasing bed voidage with increasing magnetic field intensity, as shown in Fig. 5. The bed porosity \(\varepsilon_a\), known as annular porosity, was calculated according to Hristov (2008) as follows:

\[
\varepsilon_a = 1 - \frac{V_p}{V_{bed}} = 1 - \frac{1}{[(\pi h_L \cos \alpha/12)(D_L^2 + D_L D_b + D_b^2)] \left(\frac{M_p}{\rho_s}\right)},
\]

where \(D_L = D_b + 2h_b \tan (\alpha/2)\).

As for cylindrical beds (Hristov, 1998, 2002), this trend is valid beyond the field intensity \(H_{sat}\) (Hristov, 1998), at which strong particle aggregation occurs. Beyond \(H_{sat}\), as shown in Figs. 4A and 5, aggregates continue to grow in size and orient themselves along the magnetic field lines, i.e. transverse to fluid flow. Further, the formation of stable aggregates alters the interaction between the solids inside the beds, that is, at the aggregate–aggregate, flock–flock or string–string scale as repulsion, aggregation or collision rather than at the particle–particle scale collision and aggregation only. Such extended particle aggregates are, in fact, induced magnetic bars that face each other by equal poles and repulsion between them yields a consequent increase in the depth of the entire bed, since the repulsion forces and the gas lift forces are collinear. This mechanism of string–string interaction at micro-level that affects the gross bed expansion was discussed in detail for cylindrical beds by Hristov (1998, 1999, 2002).
The above physical phenomena led to the fluidization regimes demarcated in Figs. 4 and 5. The behaviours of the solids as described above are easily detected by the pressure drop curves rather than by the collapsing (porosity evolution) curves. Even at gas flow rate slightly beyond the onset of fluidization, the pressure drop tends to decrease with increasing magnetic field intensity, while the collapsing curves exhibit detectable changes at sufficiently high gas flow rates. At gas flow rate corresponding to bubbling fluidization, application of magnetic field results in fast bed collapse without excessive formation of aggregates and the bed “freezes” easily with moderate magnetic field intensities. At sufficiently high gas flow rates high magnetic field intensities are required to stop the motion of the solids (aggregates) and to freeze the bed. The frozen bed consists of field aligned aggregates that repel each other with forces balanced by fluid drag and gravity since the bed has a fixed structure. However, the frozen bed structure is not homogeneous depending on the fluidization regime developed before applying the magnetic field. At low gas flow rates, for instance, when gas voids occur as slits, that is, frozen particle layers (plugs) separated by horizontal voids as shown in Fig. 3B, attempt to freeze an originally spouted bed results in a fixed bed with a clean central void (see Fig. 3C) as the trace of the prior spout channel is surrounded by an almost homogenously arranged annular section.
Fig. 5 and its inset show ranges of magnetic field intensity yielding the following different bed behaviours:

- Nearly linear bed collapse with increasing magnetic field intensity for relatively weak field intensities—Section A.
- Increasing bed porosity after the initial collapse, with increasing magnetic field intensity, followed eventually by final frozen bed—Section B.
- Monotonously increasing porosity with increasing magnetic field intensity—Section C.

All these trends much depend on the original bed structure before application of the magnetic field, implying that free movement of particles in the bed leads to bed collapse and aggregate formation, as shown by the initial linear section A of the collapse curves. The consequent aggregate–aggregate interaction and the aggregate formation parallel to the field lines but transverse to fluid flow result in bed expansion due to repulsive aggregate–aggregate force, as shown by section B. This fact is not new in magnetically assisted fluidization, as has been well documented by plots and physical explanations (Hristov, 2006, 2007b, 2008). For both cylindrical and tapered beds, particle–particle and aggregate–aggregate interactions are controlled by common physical mechanisms but only vessel shape affects the gross bed depth (expansion or collapse). Section C of the bed collapse curve resembles to some extent the behaviour corresponding to section B, that is, both start with unrestricted fluidization but with different degrees of solids mixing intensity. The higher the bed expansion, the easier the particle movement and aggregation, to finally yield more distinct arrangement of the aggregates along the magnetic field lines in the loose frozen bed. The latter consequently expands due to simultaneous fluid drag and aggregate-aggregate repulsion parallel to gas flow, to result in section B of the collapse curve. In this case, just beyond minimum fluidization, the particles move intensively but bed expansion is restricted to allow formation of freely moving aggregates and the particle rearrangement is manifested by the bed expansion shown as section C, followed by a plateau. Bed collapse does not occur, since the particle aggregation is enhanced by the high particle concentration just beyond minimum fluidization, which finally shifts the bed behaviour to the situation with repulsion between aggregates. However, the gas flow is not sufficient to create expanded bed to the extent for high gas flow rates such as for sections B.

4. Dimensional analysis and data correlation

This section develops dimensionless groups describing magnetically assisted tapered gas-fluidized beds employing the general expressions developed by Hristov (2008) for the magnetization-LAST mode. Dimensionless scaling of pressure drop evolution with increase in magnetic field intensity is to be correlated with experimental data. Porosity scaling is to be developed only theoretically.

4.1. General considerations and pressure drop scaling

Dimensional analysis based on the “pressure transform” approach (Hristov, 2008) generates the following dimensionless groups relevant to magnetically controlled spouted beds:

Fluid–particle interaction \( Ga = \frac{d^2 \rho_p \rho_f^2}{n_f^2} \) and \( \frac{\Delta P}{\rho_s gd_p} \); \( (2a) \)

Solids (interparticle contacts) \( Bo_{g-c} = \frac{P_c}{\rho_s gd_p} \); \( (2b) \)

Vessel geometry simplex \( \Delta D_{hl} = \frac{D_h - D_h}{2b_{ho}} = tg \left( \frac{\alpha}{2} \right) \). \( (2c) \)

These dimensionless numbers are independent variables except for \( \Delta P/\rho_p gd_p \). Generally, pressure drop varies as a function of magnetic field intensity as follows:

\[ \frac{\Delta P}{\rho_s gd_p} = f_1 \left( \frac{\Delta P}{\rho_s gd_p} \right) = f_2 \left( \frac{\Delta P}{\rho_s gd_p} \right) \]

where \( Bo_{g-m} \) is the magnetic Bond number (Hristov, 2006, 2007b, 2008). In Eq. (3) \( Ga \) characterizes the fluid–particle system, while \( \Delta D_{hl} \) represents the initial bed geometry, both not varying for conditions imposed by a given experiment. The ratio \( \rho_f U_f^2/\rho_s gd_p \) represents the initial bed conditions where the field is applied and is the initial condition of the dependent variable \( \Delta P/\rho_p gd_p \). In both dimensionless ratios the granular pressure at particle level \( \rho_s gd_p \) (Hristov, 2006, 2007b, 2008) is used as a pressure scale. The independent variable is the magnetic Bond number \( (Bo_{g-m} = \mu_0 Ms H/\rho_s gd_p) \). Hence, we may reduce the pressure drop terms in the set of dimensionless numbers by creating the ratio

\[ \left( \frac{\Delta P}{\rho_s gd_p} \right) \left( \frac{\rho_f U_f^2}{\rho_s gd_p} \right) = \frac{\Delta P}{\rho_f U_f^2} \] \( (4a) \)
which is a reasonable way to minimize the number of variables in Eq. (3) and to include the effect of the initial conditions with Magnetization-LAST mode (in fact Fluidization FIRST) represented by \((\rho_f U_f^2)\) as an initial condition (not variable).

Further, the basic phenomena such as particle aggregation and string-string interaction occur at the particle scale level so that the particle diameter \(d_p\) is the characteristic length scale of the process. In this context the Bond number \(Bo_{g-m}\) represents the stability of interparticle contact in the aggregates due to magnetic cohesion. With the above clarifications, Eq. (3) becomes

\[
\frac{\Delta P}{\rho_f U_f^2} = f \left( Ga = \frac{d_p^3 \rho_f \gamma_{fj}^2}{\eta_f^2}; \ Bo_{g-m} = \frac{\mu_0 MsH}{\rho_E g d_p}; \ \Delta D_{bl} \right). \tag{4b}
\]

In the above equation the RHS could be re-arranged in terms of two sub-functions, namely

\[
\frac{\Delta P}{\rho_f U_f^2} = f_1 \left( Ga = \frac{d_p^3 \rho_f \gamma_{fj}^2}{\eta_f^2}; \ \Delta D_{bl} \right) f_2 \left( Bo_{g-m} = \frac{\mu_0 MsH}{\rho_E g d_p} \right). \tag{4c}
\]

The initial conditions are defined by \(f_1\) which plays the role of a pre-factor of the power-law relationship (Kline, 1965; Barrenblatt, 1996), and function \(f_2\) is then represented by a power-law of the Bond number, \(Bo_{g-m}\). Hence, Eq. (4c) becomes

\[
\frac{\Delta P}{\rho_f U_f^2} = f_1 A_p \left( Bo_{g-m} = \frac{\mu_0 MsH}{\rho_E g d_p} \right)^{\eta_p}. \tag{4d}
\]

The use of \((\rho_f U_f^2)\) in the above relationships is a compromise with the tradition in spouted bed data correlations where the superficial fluid velocity at the gas inlet orifice is often used; even though this is generally incorrect since the tapered vessel has no naturally defined velocity scale (Hristov, 2008). The pressure drop in absence of a field \(\Delta P_0\) (at a given gas flow rate at which the bed is preliminarily fluidized) can also be used as a reference scale. Hence, the ratio of \((\Delta P/\Delta P_0)\) is correlated as

\[
\frac{\Delta P}{\Delta P_0} = f \left( Ga = \frac{d_p^3 \rho_f \gamma_{fj}^2}{\eta_f^2}; \ Bo_{g-m} = \frac{\mu_0 MsH}{\rho_E g d_p}; \ \Delta D_{bl} \right). \tag{5}
\]

This approach avoids the use of \((\rho_f U_f^2/\rho_E g d_p)\) as an initial condition since the tapered beds have no definitive velocity scale (see Hristov, 2008). Inasmuch as \(\Delta P_0\) and \(\Delta P_0/\rho_f U_f^2\) are interrelated through the hydrodynamics of the non-magnetized bed, the literature provides many relationships that might be used. Hence, either experimental or predicted values of \(\Delta P_0\) can be used to develop correlation such as Eq. (5). The ratio \((Q_0/Q_{mf})\) representing flow \(Q_0\) prior to bed magnetization needs to be introduced. This step was avoided in Eq. (2a), since firstly, the use of \(\Delta P_0\) violates the rule to use a common pressure scale having a natural basis, and the granular gravity pressure \(\rho_E g d_p\), in contrast to \(\Delta P_0\), is a natural pressure scale independent of experiment, and secondly, \(Q_0\) depends on experiment and the flow rate scale cannot be defined prior to experiment. Therefore, Eq. (5) is a compromise between the basic rules in data scaling and the existing practice in spouted bed data presentation. With this compromise, Eq. (5) could be simplified to

\[
\frac{\Delta P}{\Delta P_0} = f_1 A_p \left( \frac{Q_0}{Q_{mf}} \right) (Bo_{g-m})^{\eta_p}. \tag{6}
\]

4.2. Practical correlations of the pressure drop evolution with magnetic field intensity

Under the initial conditions, neither \(\Delta P/\rho_f U_f^2\) nor \(\Delta P/\Delta P_0\) is zero and it follows from Eqs. (5) and (6) that at \(H = 0\) we have \(Bo_{g-m} = 0\) and no particle aggregation exists (see limits \(Bo_{g-m} \rightarrow 0\) and \(Bo_{g-m} \rightarrow \infty\) in (Hristov, 2008)). Because of that, the function could be modified as follows:

\[
\frac{\Delta P}{\rho_f U_f^2} = A_0 + A_{\rho_0} \exp(-c_0 Bo_{g-m}), \tag{7a}
\]

\[
\frac{\Delta P}{\Delta P_0} = 1 - f_0 A_{\rho_0} \left( \frac{Q_0}{Q_{mf}} \right) (Bo_{g-m})^{\eta_p}, \tag{7b}
\]

where \(A_0 + A_{\rho_0} = 2C_D = \Delta P_0/\rho_f U_f^2\) in Eq. (7a) is the doubled friction factor of the bed in absence of a magnetic field and could be calculated independently, depending on initial conditions. The parameters of Eq. (7a), estimated through non-linear regression analysis and using Origin 6.0, are summarized in Table 1. This fitting yields scattering results, as shown in Fig. 4C, coming mainly from the fact that the LHS of Eq. (7a) does not match 1 under the initial conditions as a dimensionless variable.

More successful results were obtained with Eq. (7b) where in any case under the initial conditions of \(\Delta P/\Delta P_0 = 1\), the data

Table 1

<table>
<thead>
<tr>
<th>(Q (m^3/s) \times 10^{-3})</th>
<th>(U (m/s)) (defined by the gas inlet cross-section)</th>
<th>(Q/Q_{mf}) (–)</th>
<th>(\frac{\Delta P}{\Delta P_0}) = (A_0 + A_{\rho_0} \exp(-c_0 Bo_{g-m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.666</td>
<td>2.357</td>
<td>2.727</td>
<td>(\frac{\Delta P}{\rho_f U_f^2}) = 83.03 + 34.5 \exp(-(Bo_{g-m})8.1 \times 10^{-3}) R^2 = 0.8096; \chi^2 = 12.46; 26 data points</td>
</tr>
<tr>
<td>2.777</td>
<td>3.929</td>
<td>4.545</td>
<td>(\frac{\Delta P}{\rho_f U_f^2}) = 17.9 + 8.92 \exp(-(Bo_{g-m})12.48 \times 10^{-3}) R^2 = 0.8601; \chi^2 = 1.022; 26 data points</td>
</tr>
<tr>
<td>4.16</td>
<td>5.894</td>
<td>6.818</td>
<td>(\frac{\Delta P}{\rho_f U_f^2}) = 7.97 + 15.85 \exp(-(Bo_{g-m})1.2 \times 10^{-3}) R^2 = 0.861; \chi^2 = 1.022; 24 data points</td>
</tr>
<tr>
<td>4.999</td>
<td>7.073</td>
<td>8.182</td>
<td>(\frac{\Delta P}{\rho_f U_f^2}) = 11.75 + 56.6 \exp(-(Bo_{g-m})9 \times 10^{-3}) R^2 = 0.9505; \chi^2 = 10.29; 26 data points</td>
</tr>
</tbody>
</table>

Notes: (1) Numerical values of \(c_0\) in equations are located at end of the numerical expressions. (2) Second rows beneath the equations contain data on accuracy of correlation.
can well be approximated by a linear descending equation:

$$\Delta P = 1 - (1 \times 10^{-4}) \left( \frac{Q_0}{Q_{mf}} \right) B_{0g-m}$$

(8)

where $f_{10} = Ga\Delta D_{bl} = 9.05$, $Ga = 68.781$, $\Delta D_{bl} = 0.1316$, $n_L = 1$, and $0 \leq B_{0g-m} \leq 1665$. For all the 104 data points collected from Fig. 4A (main figure and inset) Eq. (8) fits within an accuracy of less than $10^{-4}$, scattering within the range of $-0.2$ to $+0.3$.

Alternately, the “L-shaped” pressure drop curve can be expressed by the following exponential relationship within the range of $0 \leq B_{0g-m} \leq 1665$ as shown by the inset of Fig. 4A (26 data points):

$$\Delta P = 0.245 + 0.753 \exp[-(B_{0g-m})] (5.5 \times 10^{-4}).$$

(9)

Eq (9), in fact, confirms Eq. (7a) and fits the data with an accuracy of about $10^{-3}$ in the descending section of the curve. The term $0.245$ in Eq. (9) underestimates the pressure drop at high field intensities (i.e. across the “frozen bed”), which is about 0.375 of the initial value of $\Delta P_0 \approx 4000$ Pa.

4.3. Bed collapse—general considerations about the scaling equations

This case addresses the relative bed depth ratio as function of applied field intensity:

$$\left( \frac{h}{h_U} \right) = f_h(U, d_p, H) = f_h \left( \frac{Q_0}{Q_{ms}}, Ga, B_{0g-m}; \Delta D_{bl} \right),$$

(10a)

where $h_U$ and $Q_0$ represent the initial conditions prior to bed magnetization. The bed collapse function $f_h$ could be expressed as

$$f_h(U, d_p, H) = f_{h1}(Ga; \Delta D_{bl}) f_{h2}(B_{0g-m}) \left( \frac{Q_0}{Q_{ms}} \right)^{m},$$

(10b)

where the function $f_{h1}$ and the ratio $\Delta Q = Q_0/Q_{ms}$ are, in fact, parameters since they do not vary as the bed undergoes volume shrinkage with increasing magnetic field intensity. The only independent variable is the Bond number. Actually, $f_{h1}$ is identical to $f_1$ in Eq. (4b).

Even though Eqs. (10a) and (10b) are generally correct, bed depth is not a clearly representative measure for conical bed collapse. If the function $f_h(U, d_p, H)$ is represented by the porosity ratio $\Delta \varepsilon = \varepsilon_h(Q_0)$, Eq. (7b) can be expressed as

$$\Delta \varepsilon_1 = \frac{\varepsilon}{\varepsilon_h(Q_0)} = E_1 [f_{h1}(Ga; \Delta D_{bl})]^{\varepsilon_1} [f_{h2}(B_{0g-m})]^{\varepsilon_2} \left( \frac{Q_0}{Q_{mf}} \right)^{\varepsilon_4},$$

(11a)

where $0 < \Delta \varepsilon_1 = \varepsilon_h(Q_0) \leq 1$ and $Q_0$ denotes the gas flow rate prior to bed magnetization.

Alternately, the ratio $\Delta \varepsilon$ can be expressed in terms of the initial fixed bed porosity, i.e. $\Delta \varepsilon_2 = \varepsilon/\varepsilon_0$ ($\Delta \varepsilon_2 \geq 1$), thus defining a new pre-factor and exponents in Eq. (11a):

$$\Delta \varepsilon_2 = \frac{\varepsilon}{\varepsilon_0} = E_2 [f_{h1}(Ga; \Delta D_{bl})]^{\varepsilon_2} [f_{h2}(B_{0g-m})]^{\varepsilon_2} \left( \frac{Q_0}{Q_{mf}} \right)^{\varepsilon_4}.$$  

(11b)

In both cases, $\Delta \varepsilon_1$ and $\Delta \varepsilon_2$ are decreasing functions when a bed collapses homogeneously down to the frozen state. The pre-factors $E_1$ and $E_2$ as well as the exponents $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$ and $\varepsilon_4$ have to be determined through fitting to experimental data.

For what we saw of the trends of data correlation on bed collapsing, the data available were yet not sufficient to test the scaling equations, particularly because only one single cone angle was used with a narrow size fraction of magnetite as the fluidized material. More data are needed in broader ranges of particle size and cone angles, though the principal dimensionless groups controlling the process have been tested to demonstrate the coherence of the present analysis.

5. Conclusions

Initial information about magnetically assisted gas-fluidized tapered beds with the Magnetization-LAST mode has been reported in this work. The main issue was to show the general bed behaviour represented by phase diagrams with relevant changes in pressure drop and overall bed porosity. In this context it was clearly demonstrated that:

- Magnetic field intensity increase allows change of the fluid-bed behaviour from that of a freely fluidized or spouting bed towards that of a magnetically controlled system consisting of a spouted bed topped by a progressively increasing frozen bed.
- Magnetic field intensity allows controlling both pressure drop and bed porosity thus creating conditions for suitable process performance utilizing the top located frozen bed or the internal spouted bed below.
- Dimensionless analysis draws the basic rules of the method of “pressure transforms”. Correlations of data pertinent to pressure drop collapse with increase in magnetic field intensity show adequate applications of the general scaling rules, with limited deviations, to the classic correlations in the spouted bed technology.

References


