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Application of the Variational Iteration Method for Determining the Temperature in the Heterogeneous Casting-Mould System

Edyta Hetmaniok, Konrad Kaczmarek, Damian Słota, Roman Wituła, Adam Zielonka

Abstract – *In this paper an application of the variational iteration method for determining the temperature in the heterogeneous casting-mould system is presented. Considered problem is modeled by means of the system of two heat conduction equations – for the casting and for the mould, on the contact surface of which the boundary condition of the fourth kind with non-zero thermal resistance is defined. An example illustrating the discussed application and confirming usefulness of this method in solving such kind of problems is also presented.* ***Copyright © 2012 Praise Worthy Prize S.r.l. - All rights reserved.***

***Keywords**: Heat Conduction, Casting-Mould System, Variational Iteration Method*

Nomenclature

Roman symbols

D_1 Region of casting
D_2 Region of mould
k_i Thermal conductivity [W/mk]
L Linear operator
N Nonlinear operator
R Thermal resistance [m^{2}k/W]
t Time [s]
u Temperature in region D_1 [k]
$\tilde{u}$ Restricted variation
v Temperature in region D_2 [k]
x Spatial location [m]

Greek

α_i Thermal diffusivity [m^2/s]
Γ_i Components of boundary
λ General Lagrange multiplier

I. Introduction

The variational iteration method was developed by Ji-Huan He [1]-[5]. By using this method we are able to solve the nonlinear equation:

$$L\big(u(t)\big) + N(u(t)) = f(t) \tag{1}$$

where L denotes the linear operator, N is the nonlinear operator, f refers to a known function and u is a sought function. At first, we construct a correction functional of the form:

$$u_n(t) = u_{n-1}(t) + \\ + \int_0^t \lambda(s)\Big(L\big(u_{n-1}(s)\big) + N\big(\tilde{u}_{n-1}(s)\big) - f(s)\Big)\,ds \tag{2}$$

where $\tilde{u}_{n-1}$ is a restricted variation, $\lambda(s)$ denotes a general Lagrange multiplier which can be identified optimally by the variational theory and $u_0(s)$ is an initial approximation. Next, we determine the general Lagrange multiplier and identify it as a function of $\lambda = \lambda(s)$.

Finally, we obtain the iteration formula

$$u_n(t) = u_{n-1}(t) + \\ + \int_0^t \lambda(s)\Big(L\big(u_{n-1}(s)\big) + N\big(u_{n-1}(s)\big) - f(s)\Big)\,ds \tag{3}$$

from which an approximate solution (and frequently an exact solution as well) of Eq. (1) may be derived.

Variational iteration method is a useful and efficient tool for solving a wide class of nonlinear operator equations. For example, there are available publications describing application of this method for investigating mathematical models appearing in biology [6] and astrophysics [7]. The use of discussed method to direct and inverse Stefan problems is considered in papers [8]-[11]. In [12] the variational iteration method is applied for finding the solution of Laplace equation, in [13],[14] for solving the heat transfer equation and wave equation, in [15] for determining the solution of hyperbolic differential equations and in [16] for solving the systems of partial differential equations. Variational iteration method is also applied for finding the solution of fractional equation [17],[18] (for solving equations of that kind the heat-balance integral method is often used as well [19]-[21]). Convergence of the variational iteration method is discussed in papers [22]-[26].

In this paper and application of variational iteration method for determining the temperature distribution in the heterogeneous casting-mould system, on the contact surface of which the thermal resistance appears, is presented. Discussed problem is modeled with the aid of

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the system of two heat conduction equations - for the casting and for the mould, in the contact points of which the boundary condition of the fourth kind with non-zero thermal resistance is defined.

Application of this method for similar problem, but with the perfect thermal contact between casting and mould (no thermal resistance) is presented in paper [27].

II. Statement of the problem

Let us consider two regions D_1 (casting) and D_2 (mould) (see Figure 1). On boundary of these domains five components $\Gamma_i, i = 1, 2, \ldots, 5$, are distributed, where the initial and boundary conditions are given (see Fig. 1).

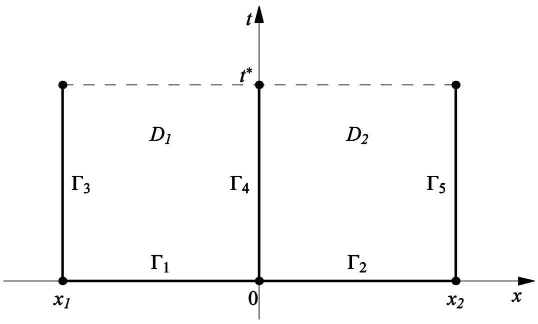

Fig. 1. Domain of the problem

In domains D_1 and D_2 the following heat conduction equations are considered:

$$\alpha_u \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u}{\partial t}(x,t) \qquad (x,t) \in D_1 \tag{4}$$

$$\alpha_v \frac{\partial^2 v(x,t)}{\partial x^2} = \frac{\partial v}{\partial t}(x,t) \qquad (x,t) \in D_2 \tag{5}$$

with initial conditions on boundaries Γ_1 and Γ_2:

$$u(x,0) = \varphi_u(x), \qquad x \in [x_1, 0] \tag{6}$$

$$v(x,0) = \varphi_v(x), \qquad x \in [0, x_2] \tag{7}$$

and Dirichlet's conditions on boundaries Γ_3 and Γ_5:

$$u(x_1,t) = \psi_u(t), \qquad t \in [0, t^*) \tag{8}$$

$$v(x_2,t) = \psi_v(t), \qquad t \in [0, t^*) \tag{9}$$

On common boundary Γ_4 the boundary conditions of the fourth kind with the non-zero thermal resistance are given:

$$-k_u \frac{\partial u(x,t)}{\partial x}\bigg|_{x=0} = \frac{v(0,t) - u(0,t)}{R} = = -k_v \frac{\partial v(x,t)}{\partial x}\bigg|_{x=0}, \quad t \epsilon [0, t^*) \tag{10}$$

where u and v denote the temperature, α_u and α_v are the thermal diffusivity, k_u and k_v describe the thermal conductivity, R denotes the thermal resistance and t and x refer to the time and spatial location, respectively. We shall seek the functions $u(x,t)$ and $v(x,t)$ determined in domains D_1 and D_2, respectively, satisfying the proper heat conduction equations and the above specified conditions.

III. Solution of the Problem

For solving the problem described in previous section we apply the variational iteration method. We will use two approaches based on applying the correction functionals in *t*-direction and in *x*-direction, respectively.

Method 1. The correction functionals in *t*-direction for equations (4) and (5) can be expressed as follows:

$$u_{n+1}(x,t) = u_n(x,t) + + \int_0^t \lambda_1(s) \left(\frac{\partial u_n(x,s)}{\partial s} - \alpha_u \frac{\partial^2 \tilde{u}_n(x,s)}{\partial x^2} \right) ds \tag{11}$$

$$v_{n+1}(x,t) = v_n(x,t) + + \int_0^t \lambda_2(s) \left(\frac{\partial v_n(x,s)}{\partial s} - \alpha_v \frac{\partial^2 \tilde{v}_n(x,s)}{\partial x^2} \right) ds \tag{12}$$

where $\tilde{u}_n$ and $\tilde{v}_n$ denote the restricted variations and $\lambda_1$and λ_2 are the general Lagrange multipliers which can be identified optimally by the variational theory. The stationary conditions are given by:

$$\lambda'_1(s) = 0, \quad (1 + \lambda_1(s))_{s=t} = 0 \tag{13}$$

$$\lambda'_2(s) = 0, \quad (1 + \lambda_2(s))_{s=t} = 0 \tag{14}$$

so that:

$$\lambda_1(s) = -1 \tag{15}$$

$$\lambda_2(s) = -1 \tag{16}$$

Hence, we obtain the following iteration formula:

$$u_{n+1}(x,t) = u_n(x,t) + - \int_0^t \left(\frac{\partial u_n(x,s)}{\partial s} - \alpha_u \frac{\partial^2 u_n(x,s)}{\partial x^2} \right) ds \tag{17}$$

$$v_{n+1}(x,t) = v_n(x,t) + - \int_0^t \left(\frac{\partial v_n(x,s)}{\partial s} - \alpha_v \frac{\partial^2 v_n(x,s)}{\partial x^2} \right) ds \tag{18}$$

As the initial approximations $u_0(x,t)$ and $v_0(x,t)$ we choose the functions describing the initial conditions.

Method 2. In this approach we apply for equations (4) and (5) the correction functionals in x-direction having the formula:

$$u_{n+1}(x,t) = u_n(x,t) + + \int_x^0 \lambda_1(s)\left(\frac{\partial^2 u_n(s,t)}{\partial s^2} - \frac{1}{\alpha_u}\frac{\partial \tilde{u}_n(s,t)}{\partial t}\right) ds \quad (19)$$

$$v_{n+1}(x,t) = v_n(x,t) + + \int_0^x \lambda_2(s)\left(\frac{\partial^2 v_n(s,t)}{\partial s^2} - \frac{1}{\alpha_v}\frac{\partial \tilde{v}_n(s,t)}{\partial t}\right) ds \quad (20)$$

where $\tilde{u}_n$ and $\tilde{v}_n$ denote, as before, the restricted variations and λ_1 and λ_2 are the general Lagrange multipliers. From equations (19) and (20) the general Lagrange multipliers can be identified as follows:

$$\lambda_1(s) = x - s \quad (21)$$

$$\lambda_2(s) = s - x \quad (22)$$

Substituting obtained values of λ_1 and λ_2 into Eqs. (19) and (20) we receive the following iteration formula:

$$u_{n+1}(x,t) = u_n(x,t) + + \int_x^0 (x-s)\left(\frac{\partial^2 u_n(s,t)}{\partial s^2} - \frac{1}{\alpha_u}\frac{\partial u_n(s,t)}{\partial t}\right) ds \quad (23)$$

$$v_{n+1}(x,t) = v_n(x,t) + + \int_0^x (s-x)\left(\frac{\partial^2 v_n(s,t)}{\partial s^2} - \frac{1}{\alpha_v}\frac{\partial v_n(s,t)}{\partial t}\right) ds \quad (24)$$

Next, we choose the initial approximations in the following form:

$$u_0(x,t) = A_u + B_u x \quad (25)$$

$$v_0(x,t) = A_v + B_v x \quad (26)$$

where A_u, A_v, B_u and B_v are parameters independent from variable x. By employing condition (10) we establish that parameters A_u, A_v, B_u and B_v must comply with the following relations:

$$A_v - A_u = -R\, k_u B_u \quad (27)$$

$$B_v = \frac{k_u}{k_v} B_u \quad (28)$$

IV. Examples

Theoretical considerations introduced in the previous sections will be illustrated now with some examples.

Example 1. At first, we consider the example in which we take:

$x_1 = -1$, $x_2 = 1$, $\alpha_u = \frac{1}{4}$, $\alpha_v = 1$, $k_u = 1$, $k_v = 2$ and $R = e^{-t}$, $\varphi_u(x) = e^{2x} + 2$, $\varphi_v(x) = e^x$, $\psi_u(t) = e^{t-2} + 2$ and $\psi_v(t) = e^{t+1}$

Starting with initial approximations in the form:

$$u_0(x,t) = e^{t-2} + 2$$
$$v_0(x,t) = e^{t+1}$$

and using the first method (formulae (17) and (18)) we can obtain the following results:

$$u_1(x,t) = 2 + e^{2x}(1+t)$$
$$u_2(x,t) = 2 + e^{2x}\left(1 + t + \frac{t^2}{2}\right)$$
$$u_3(x,t) = 2 + e^{2x}\left(1 + t + \frac{t^2}{2} + \frac{t^3}{6}\right)$$
$$u_4(x,t) = 2 + e^{2x}\left(1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \frac{t^4}{24}\right)$$
$$\vdots$$

and:

$$v_1(x,t) = e^x(1+t)$$
$$v_2(x,t) = e^x\left(1 + t + \frac{t^2}{2}\right)$$
$$v_3(x,t) = e^x\left(1 + t + \frac{t^2}{2} + \frac{t^3}{6}\right)$$
$$v_4(x,t) = e^x\left(1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \frac{t^4}{24}\right)$$
$$\vdots$$

In general we receive:

$$u_n(x,t) = 2 + e^{2x}\sum_{k=0}^{n}\frac{t^k}{k!}$$
$$v_n(x,t) = e^x\sum_{k=0}^{n}\frac{t^k}{k!}$$

Therefore we get:

$$u(x,t) = \lim_{n\to\infty} u_n(x,t) = 2 + e^{2x+t} \quad (29)$$

$$v(x,t) = \lim_{n\to\infty} v_n(x,t) = e^{x+t} \quad (30)$$

which are the exact solutions of discussed example.

Example 2. Now we consider the example in which we set:

$x_1 = -1$, $x_2 = 1$, $\alpha_u = 1$, $\alpha_v = 1$, $k_u = 4$, $k_v = 3$ and $R = \frac{7}{3}e^{-t}$, $\varphi_u(x) = -\frac{3}{7}e^{-x} + 6$, $\varphi_v(x) = \frac{1}{14}(e^x - 7e^{-x})$, $\psi_u(t) = -\frac{3}{7}e^{t+1} + 6$ and $\psi_v(t) = \frac{1}{14}(e^{t+1} - 7e^{t-1}) + 2$

Starting with initial approximations in the form:

$$u_0(x,t) = -\frac{3}{7}e^{-x} + 6 \quad (31)$$

$$v_0(x,t) = \frac{1}{14}(e^x - 7e^{-x}) + 2 \quad (32)$$

and applying the first method we can obtain the following results:

$$u_1(x,t) = 6 + e^{-x}\left(-\frac{3}{7} - \frac{3t}{7}\right)$$
$$u_2(x,t) = 6 + e^{-x}\left(-\frac{3}{7} - \frac{3t}{7} - \frac{3t^2}{14}\right)$$
$$u_3(x,t) = 6 + e^{-x}\left(-\frac{3}{7} - \frac{3t}{7} - \frac{3t^2}{14} - \frac{t^3}{14}\right)$$
$$u_4(x,t) = 6 + e^{-x}\left(-\frac{3}{7} - \frac{3t}{7} - \frac{3t^2}{14} - \frac{t^3}{14} - \frac{t^4}{56}\right)$$
$$\vdots$$

and:

$$v_1(x,t) = 2 + e^{-x}\left(-\frac{1}{2} - \frac{t}{2}\right) + e^{x}\left(\frac{1}{14} + \frac{t}{14}\right)$$
$$v_2(x,t) = 2 + e^{-x}\left(-\frac{1}{2} - \frac{t}{2} - \frac{t^2}{4}\right) + \\ + e^{x}\left(\frac{1}{14} + \frac{t}{14} + \frac{t^2}{28}\right)$$
$$v_3(x,t) = 2 + e^{-x}\left(-\frac{1}{2} - \frac{t}{2} - \frac{t^2}{4} - \frac{t^3}{12}\right) + \\ + e^{x}\left(\frac{1}{14} + \frac{t}{14} + \frac{t^2}{28} + \frac{t^3}{84}\right)$$
$$v_4(x,t) = 2 + e^{-x}\left(-\frac{1}{2} - \frac{t}{2} - \frac{t^2}{4} - \frac{t^3}{12} - \frac{t^4}{48}\right) + \\ + e^{x}\left(\frac{1}{14} + \frac{t}{14} + \frac{t^2}{28} + \frac{t^3}{84} + \frac{t^4}{336}\right)$$
$$\vdots$$

and in general:

$$u_n(x,t) = 6 - \frac{3}{7}e^{-x}\sum_{k=0}^{n}\frac{t^k}{k!}$$
$$v_n(x,t) = 2 - \frac{1}{2}e^{-x}\sum_{k=0}^{n}\frac{t^k}{k!} + \frac{1}{14}e^{x}\sum_{k=0}^{n}\frac{t^k}{k!}$$

Thus we have:

$$u(x,t) = \lim_{n\to\infty} u_n(x,t) = 6 - \frac{3e^{t-x}}{7} \tag{33}$$

$$v(x,t) = \lim_{n\to\infty} v_n(x,t) = 2 - \frac{e^{t-x}}{2} + \frac{e^{t+x}}{14} \tag{34}$$

which are the exact solutions.

Example 3. In this example the assumed data are the same as in Example 2, but unlike previously, for determining the solution the second method will be used.

The initial approximations are given by the functions:

$$A_u(t) = 6 - \frac{3e^t}{7}, \qquad B_u(t) = \frac{3e^t}{7}$$
$$A_v(t) = 2 - \frac{3e^t}{7}, \qquad B_v(t) = \frac{4e^t}{7}$$

Using the second method (formulae (23) and (24)) we can obtain the results given below:

$$u_1(x,t) = 6 + e^{t}\left(-\frac{3}{7} + \frac{3x}{7} - \frac{3x^2}{14} + \frac{x^3}{14}\right)$$
$$u_2(x,t) = 6 + e^{t}\left(\begin{array}{c} -\frac{3}{7} + \frac{3x}{7} - \frac{3x^2}{14} + \\ + \frac{x^3}{14} - \frac{x^4}{56} + \frac{x^5}{280} \end{array}\right)$$
$$u_3(x,t) = 6 + e^{t}\left(\begin{array}{c} -\frac{3}{7} + \frac{3x}{7} - \frac{3x^2}{14} + \frac{x^3}{14} + \\ -\frac{x^4}{56} + \frac{x^5}{280} - \frac{x^6}{1680} + \frac{x^7}{11760} \end{array}\right)$$
$$\vdots$$

and:

$$v_1(x,t) = 2 + e^{t}\left(-\frac{3}{7} + \frac{4x}{7} - \frac{3x^2}{14} + \frac{2x^3}{21}\right)$$
$$v_2(x,t) = 2 + e^{t}\left(\begin{array}{c} -\frac{3}{7} + \frac{4x}{7} - \frac{3x^2}{14} + \\ + \frac{2x^3}{21} - \frac{x^4}{56} + \frac{x^5}{210} \end{array}\right)$$
$$v_3(x,t) = 2 + e^{t}\left(\begin{array}{c} -\frac{3}{7} + \frac{4x}{7} - \frac{3x^2}{14} + \frac{2x^3}{21} + \\ -\frac{x^4}{56} + \frac{x^5}{210} - \frac{x^6}{1680} + \frac{x^7}{8820} \end{array}\right)$$
$$\vdots$$

and in general:

$$u_n(x,t) = 6 + \frac{3}{7}e^{t}\left(\sum_{i=0}^{n}\frac{x^{2k+1}}{(2k+1)!} - \sum_{i=0}^{n}\frac{x^{2k}}{(2k)!}\right)$$

$$v_n(x,t) = 2 + e^{t}\left(\frac{4}{7}\sum_{i=0}^{n}\frac{x^{2k+1}}{(2k+1)!} - \frac{3}{7}\sum_{i=0}^{n}\frac{x^{2k}}{(2k)!}\right)$$

Hence, after some transformations the exact solutions are derived:

$$u(x,t) = 6 + \frac{3}{7}e^{t}(\sinh x - \cosh x) = 6 - \frac{3e^{t-x}}{7} \tag{35}$$

$$v(x,t) = 2 + e^{t}\left(\frac{4}{7}\sinh x - \frac{3}{7}\cosh x\right) = \\ = 2 - \frac{e^{t-x}}{2} + \frac{e^{t+x}}{14} \tag{36}$$

V. Conclusion

In this paper we have described the application of variational iteration method for determining the temperature in the heterogeneous casting-mould system, on the contact surface of which the thermal resistance appears. Advantage of this method is the fact that discretization of considered region is not required, in contrast with classical methods, like the finite difference method or finite element method. In result of using the variational iteration method we obtain the sequence of approximations convergent to the exact solution if it exists. In each considered example we succeeded in finding the exact solution. In case when determining the general form of functions u_n and v_n is not possible, one can execute some number of iterations and obtain the approximate solution in this way.

References

[1] J.-H. He, Approximate analytical solution for seepage flow with fractional derivatives in porous media, *Comput. Methods Appl. Mech. Engrg. 167 (1998) 57–68*.

[2] J.-H. He, Approximate solution of nonlinear differential equations with convolution product nonlinearities, *Comput. Methods Appl. Mech. Engrg. 167 (1998) 69–73.*

[3] J.-H. He, Variational iteration method – a kind of non-linear analytical technique: some examples, *Int. J. Non-Linear Mech. 34 (1999) 699–708.*

[4] J.-H. He, *Non-Perturbative Methods for Strongly Nonlinear Problems* (Dissertation.de-Verlag, 2006).

[5] J.-H. He, G.-C. Wu, F. Austin, The variational iteration method which should be followed, *Nonlinear Sci. Lett. A 1 (2010) 1–30.*

[6] F. Shakeri, M. Dehghan, Solution of a model describing biological species living together using the variational iteration method, *Math. Comput. Modelling 48 (2008) 685–699.*

[7] M. Dehghan, F. Shakeri, Approximate solution of a differential equation arising in astrophysics using the variational iteration method, *New Astronomy 13 (2008) 53–59.*

[8] E. Hetmaniok, D. Słota, A. Zielonka, Solution of the solidification problem by using the variational iteration method, *Arch. Foundry Eng. 9 (4) (2009) 63–68.*

[9] D. Słota, Direct and inverse one-phase Stefan problem solved by variational iteration method, *Comput. Math. Appl. 54 (2007) 1139–1146.*

[10] D. Słota, A. Zielonka, A new application of He's variational iteration method for the solution of the one-phase Stefan problem, *Comput. Math. Appl. 58 (2009) 2489–2494.*

[11] E. Hetmaniok, D. Słota, R. Wituła, A. Zielonka, Comparison of the Adomian decomposition method and the variational iteration method in solving the moving boundary problem, *Comput. Math. Appl. 61 (2011) 1931–1934.*

[12] A.-M. Wazwaz, The variational iteration method for exact solutions of Laplace equation, *Phys. Lett. A 363 (2007) 260–262.*

[13] M. Miansari, D. D. Ganji, M. Miansari, Application of He's variational iteration method to nonlinear heat transfer equations, *Phys. Lett. A 372 (2008) 779–785.*

[14] J. Biazar, H. Ghazvini, An analytical approximation to the solution of a wave equation by a variational iteration method, *Appl. Math. Lett. 21 (2008) 780–785.*

[15] J. Biazar, H. Ghazvini, He's variational iteration method for solving hyperbolic differential equations, *Int. J. Nonlin. Sci. Numer. Simulat. 8 (2007) 311–314.*

[16] A.-M. Wazwaz, The variational iteration method for solving linear and nonlinear systems of PDEs, *Comput. Math. Appl. 54 (2007) 895–902.*

[17] S. Das, Analytical solution of a fractional diffusion equation by variational iteration method, *Comput. Math. Appl. 57 (2009) 483–487.*

[18] S. Yang, A. Xiao, H. Su, Convergence of the variational iteration method for solving multi-order fractional differential equations, *Comput. Math. Appl. 60 (2010) 2871–2879.*

[19] J. Hristov, Heat-balance integral to fractional (half-time) heat diffusion sub-model, *Thermal Science 14 (2010) 291–316.*

[20] J. Hristov, Transient flow of a generalized second grade fluid due to a constant surface shear stress: an approximate integral-balance solution, *Int. Rev. Chem. Eng. 3 (2011) 802–809.*

[21] J. Hristov, Starting radial subdiffusion from a central point through a diverging medium (a sphere): heat-balance integral method, *Thermal Science 15 suppl. 1 (2011) s5–s20.*

[22] M. Tatari, M. Dehghan, On the convergence of He's variational iteration method, *J. Comput. Appl. Math. 207 (2007) 121–128.*

[23] D. K. Salkuyeh, Convergence of the variational iteration method for solving linear systems of ODEs with constant coefficients, *Comput. Math. Appl. 56 (2008) 2027–2033.*

[24] R. Saadati, M. Dehghan, S. M. Vaezpour, M. Saravi, The convergence of he's variational iteration method for solving integral equations, *Comput. Math. Appl. 58 (2009) 2167–2171.*

[25] Z. M. Odibat, A study on the convergence of variational iteration method, *Math. Comput. Modelling 51 (2010) 1181–1192.*

[26] D. K. Salkuyeh, H. R. Ghehsareh, Convergence of the variational iteration method for the telegraph equation with integral conditions, *Numer. Methods Partial Differential Eq. 27 (2011) 1442–1455.*

[27] D. Słota, Exact solution of the heat equation with boundary condition of the fourth kind by He's variational iteration method, *Comput. Math. Appl. 58 (2009) 2495–2503.*

Authors' information

Edyta Hetmaniok is an assistant professor in Institute of Mathematics, Silesian University of Technology, Gliwice, Poland. She works on numerical methods for direct and inverse problems connected with the heat conduction and on other applications of mathematics. Moreover, she is engaged in research on elements of the number theory and some related problems.
E-mail: edyta.hetmaniok@polsl.pl

Konrad Kaczmarek is an assistant professor in Institute of Mathematics, Silesian University of Technology, Gliwice, Poland. He works on mathematical modeling of technical problems, in particular by applying genetic algorithms and neural networks.
E-mail: konrad.kaczmarek@polsl.pl

Damian Słota is an associate professor in Institute of Mathematics, Silesian University of Technology, Gliwice, Poland. He is engaged in investigating numerical methods for direct and inverse problems connected with the heat conduction and in other applications of mathematics. Moreover, he works on elements of the number theory and some related problems.
E-mail: damian.slota@polsl.pl

Roman Wituła is an assistant professor in Institute of Mathematics, Silesian University of Technology, Gliwice, Poland. He investigates various problems concerning algebra, number theory, special functions and their applications in technical sciences.
E-mail: roman.witula@polsl.pl

Adam Zielonka is an assistant professor in Institute of Mathematics, Silesian University of Technology, Gliwice, Poland. He works on numerical methods for direct and inverse problems connected with the heat conduction and on other applications of mathematics.
E-mail: adam.zielonka@polsl.pl