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Thermoeconomical Optimization of Energy Intensive Systems on Graphs

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Abstract – *In the design and operation of energy intensive systems the problem of improving the efficiency is very important. The main way for solving this problem is optimization. This paper describes the general approach for thermoeconomical optimization systems with linear structure as well as systems with pair interplay of flows.*
The suggested method is based on building and analysis of special graphs of thermoeconomical expenditure. The method is illustrated by an example of a system optimization for thermal treatment of chlorine water as well as a numerical example of a heat exchanger network optimization with one-time interaction of six-flows. **Copyright © 2010 Praise Worthy Prize S.r.l. - All rights reserved.**

Keywords: *Optimization, Exergy, Thermoeconomics, Graphs*

I. Introduction

Processes that take place in complex energy intensive systems are characterized by the mutual transformation of quantitatively different power resources. The fast growth and development of modern technologies requests a thermodynamic analysis and optimization of such systems, based on the combined application of both laws of thermodynamics, and demands an exergy approach [1,2]. Exergetic methods are universal and make it possible to estimate the energy fluxes and to develop energy balances for every element of the system using a common criterion of efficiency. Therefore, the exergetic methods are meaningful in analysis and calculations.

Despite its usefulness, the benefits of the exergetic approach were not fully realized until recent years.

One reason for this situation is the underestimation of exergetic functions for mathematical modeling, synthesis, and optimization of flow sheets. Another reason is the mathematical difficulty of the exergetic approach in thermodynamic analysis. Meanwhile, the increasing complexity of optimization problems requires more effective and powerful mathematical methods. Hence, during the last few years, many papers with different applications of exergetic methods have been published [3-7].

The above referenced papers, as well as the author's earlier investigations [8-13] show that one of the most effective mathematical methods used for exergetic analysis and optimization is the method of graph theory [14,15].

The benefit of graph models can also be demonstrated by its flexibility and its wide variety of possible applications.

Possible exergy topological methods include the sole use or combination of exergy flow graphs, exergy loss graphs, and thermoeconomical graphs [3-7].

Systems with linear structure as well as systems with pair interplay of flow are often used in energy technology and in other branches of industry. For that reason it is necessary to study the problem of these systems optimization separately from the systems of arbitrary structure.

II. Method Optimization of Linear System

First, let us consider a homogeneous system that contains n different elements of one type (as shown in Fig. 1).

In this system one flow h_j, j=1 interacts successively with flows C_i , i=1,2,…n.

In the problem of optimal synthesis, this can be reformulated in the following form:

it is necessary to distribute the multitude of flows

$$C = \{C_1,C_2,...,C_i,...,C_n\} \tag{1}$$

along the flow h_j (j=1) and in result of interaction of which parameters of flow h_j in outlet of system will be in interval of required constrains and thermoeconomical criteria will be minimized:

$$\sum_i \sum_j Z_{ij} = Z_\Sigma^{min} \tag{2}$$

where Z_{ij} -thermoeconomical expenditure at i-element (j=1).

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For solving this type of problem, it is necessary to build the graph of thermoeconomical expenditure as it was shown in [8].

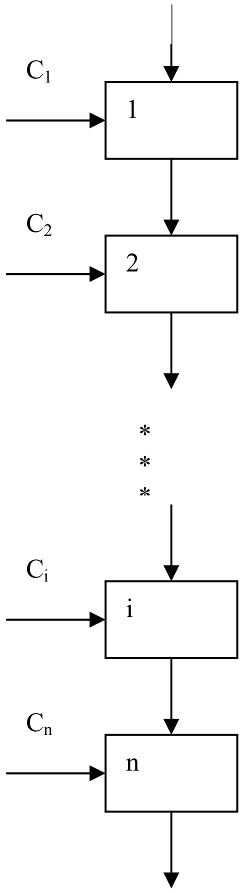

Fig. 1. Linear system

In our case this graph will be a tree Z = (N,D) multitude of nodes N displays the possibility of distribution of flows in the system, the multitude of arcs D, displays the possible meanings of thermoeconomical expenditures.

The governing equation which are representing these levels are:

$$N_p = \left\{ C_1^{(p)}, C_2^{(p)}, ..., C_{i_p}^{(p)}, ..., C_{[n-(p-1)]}^{(p)} \right\}$$
$$p = 1,2,...,k; i_p = 1,2,...,\left[n-\left(p-1\right)\right] \quad (3)$$

where:

$$N_p \subset C, p = 1,2,...,k,$$
$$p = 0 \Rightarrow \left|N_0\right| = 1$$
$$p = 1 \Rightarrow \left|N_p\right| = \left|C\right|, C - N_p = \varnothing \quad (4)$$
$$1 \le p \le k \Rightarrow \left|N_p\right| \le \left|C\right|$$

$$\forall \left(C_{i_{p-1}}^{(p-1)}, C_{i_p}^{(p)} \right) \in D \Rightarrow \left(C_{i_{p-1}}^{(p-1)}, C_{i_p}^{(p)} \right) = Z_{i_p}^{(p)}$$
$$\forall \left(C_{i_{p-1}}^{(p-1)}, C_{i_p}^{(p)} \right) \notin D \Rightarrow \left(C_{i_{p-1}}^{(p-1)}, C_{i_p}^{(p)} \right) = \infty \quad (5)$$

where symbol ∞ shows that arcs of this type are absent. The flow h_j in graph Z = (N,D) is described as node $C_0^{(0)}$. Then for obtaining conditions (2) it is necessary to find an optimal way:

$$\bar{C} = \left(C_0^{(0)}, C_1^{(1)}, ..., C_{i_p}^{(p)}, ..., C_{[n-(p-1)]}^{(k)} \right)$$
$$\bar{C} \subset N \quad (6)$$

so that:

$$\sum_{i_p} \sum_{p} Z_{i_p}^{(p)} = Z_{\Sigma}^{min} \quad (7)$$

The algorithm of Belmann-Kalaba is usually used for seeking the optimal way in graphs without contours. This algorithm is based on matrix of thermoeconomical expenditure [8,9].

In our case the graph of thermoeconomical expenditure is successive:

$$\Gamma_p N_p = N_{p+1} \quad (8)$$

where Γ_p is display of set N_p, and condition of Eq. (5) will be valid for elements of matrix (see Fig. 3) which are located in the intersection of columns $C_{i_p}^{(p)}$ and lines:

$$C_{i_{p-1}}^{(p-1)}, p = 1,2,...,k; \quad i_p = 1,2,...,\left[n-\left(p-1\right)\right]$$

This feature of graph of thermoeconomical expenditure allows one to simplify the matrix of expenditure and to reduce the number of analyzed variant in n times [8,9].

Based on features of the thermoeconomical expenditure graph, we recommend the algorithm of searching optimal variant be used.

Each step of seeking an optimal variant is successively compared with thermoeconomical expenditure $Z_{i_p}^{(p)}$ and $Z_{min}^{(p)}$. In result, the flow corresponded equation (7) can be found. Then applying the procedure of seeking $Z_{min}^{(p)}$ to all k steps, we will find the optimal flow distribution which corresponds to the condition in Eq. (2).

In case of inhomogeneous linear systems optimization, the main idea of this approach will retain.

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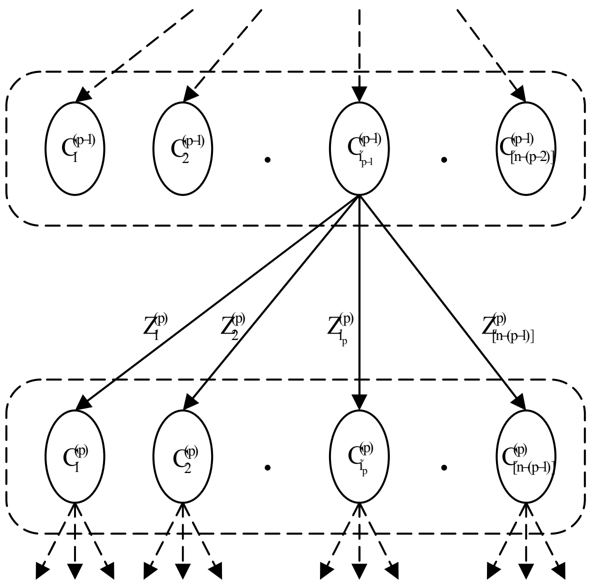

Fig. 2. (P-1) and P - levels tree of thermoeconomical expenditure

Since inhomogeneous elements are able to change the different characteristics of flow h_j, it is necessary to consider not only the p th-step but also previous steps of the system's optimization. Consequently the method of dynamic programming has to be changed to branch and bound method. On this approach at each step we seek and then save expenditure $Z_{\Sigma}^{(p)min}$, where $Z_{\Sigma}^{(p)}$ is the sum of thermoeconomical expenditure for all p steps of considered variant. Then expenditure $Z_{\Sigma}^{(p)min}$ will be compared with the analogous sums for the previous steps $(p-1),(p-2),...,1$.

Then the variant corresponding to the following equation has to be developed:

$$Z_{\Sigma}^{min} = min\left[Z_{\Sigma}^{(l)min} \right], \; l = 1,2,...,p \qquad (9)$$

Then elements for the next step of optimization have to be taken from the multitude N_{p+1}, which corresponds to equation (8).

III. Optimal Synthesis of Chlorine Water Refrigeration

The systems of chlorine water refrigeration usually have a linear structure and can be describe by Fig. 1. In this case the flow h_1 will display the flow of refrigerated chlorine water and C is the set of flows which refrigerate the flow h_1. The set of flows C include flows of industrial water and cooled water. If industrial water is used, the surface of heat exchangers and appropriate expenditures will be bigger than in the case of using cooled water. But using the cooled water requires the additional expenditures for its cooling. So the question is to find the variant of system of chlorine water refrigeration with minimum expenditure:

$$Z_{\Sigma} = \sum_{i=1}^{n} Z_i^{min} \qquad (10)$$

$$Z_i^{min} = min\left\{ \left(Z_i^A + Z_i^{iw} \right), \left(Z_i^A + Z_i^{cw} \right) \right\} \qquad (11)$$

where:

- Z_i^A is the year investments cost (for surface, repairing, salary of personal etc.) for heat exchanger i**,**
- Z_i^{iw} is the year expenditure for industrial water
- Z_i^{cw} is the year expenditure for cooled water

$$Z_i^{cw} = P_i^{cw} E_i^{cw} \tau$$

where:

- P_i^{cw} is the price of one kJ cooled water
- E_i^{cw} is the exergy of cooled water which can becalculated as exergy of a heat flow entered to heat exchanger **i.**
- τ is the period work of the system during a year

$$Z_i^{iw} = P_i^{iw} m_i^{iw} \tau \qquad (12)$$

Here:

- P_i^{iw} is the price of one kilogram of industrial water (as the differ between parameters of industrial water and environment are very little we can assume that exergy of industrial water equal zero) for heat exchanger **i.**
- m_i^{iw} is the mass flow of industrial water for heat exchanger **i.**

As an example, a typical line of chlorine production was taken (the mass flow of chlorine is 2.53 kg/s) with mass flow of chlorine water 10.9 kg/s. The working time during a year is τ=7200 hours. The temperature of chlorine water at inlet of system is 50°C the required temperature at outlet of the system is 15°C. The heat exchangers for such scheme are titanium refrigerators

with a surface area each of 60m^2 with a heat transfer coefficient 700 W/(m^2 K). The initial temperature of the industrial water is 20°C. The initial temperature of cooled water is 5°C. The year investments cost for the heat exchanger is 0.0666 USD for one square meter of surface. Price of exergy of cooled water is 0.0038 USD/MJ, price of industrial water is 0.065x10^{-6} USD/kg.

Application of the procedure described above for optimisation of this system gives the tree of solution shown in Fig. 4. The left branch of the tree displays variants of using cooled water, and the right branch-

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industrial water. Each level of tree (excluding level zero) has two nodes with appropriate temperature of chlorine water and thermoeconomical expenditure. For further developing of tree on each level taken (as it was described above), the node with a minimum of Z_{Σ}.

It is seen that on the first three levels, is chosen the variant of using industrial water and only on last one– cooled water. So optimal system of chlorine water refrigeration will include three heat exchangers with industrial water and one with cooled water.

The optimal meaning of thermoeconomical expenditure for this system is 3559 USD per year.

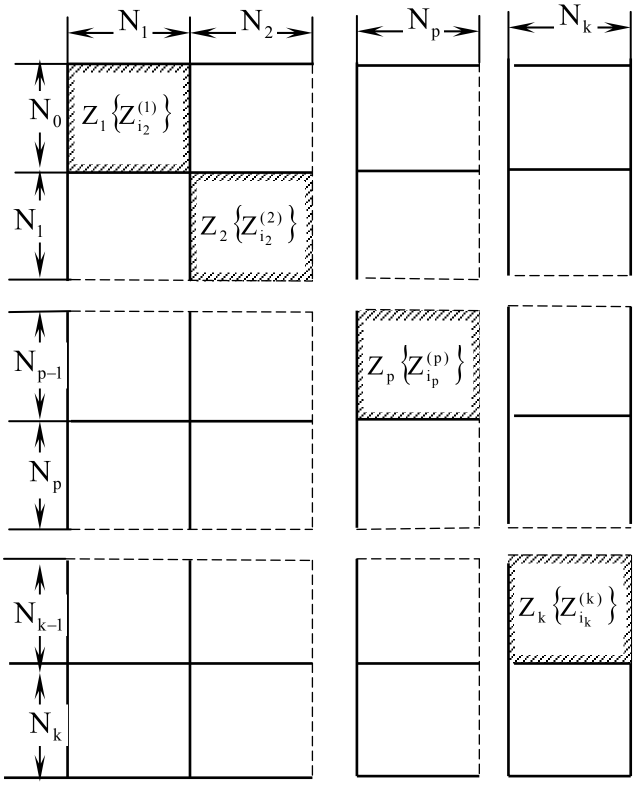

Fig. 3. Matrix of thermoeconomical expenditure

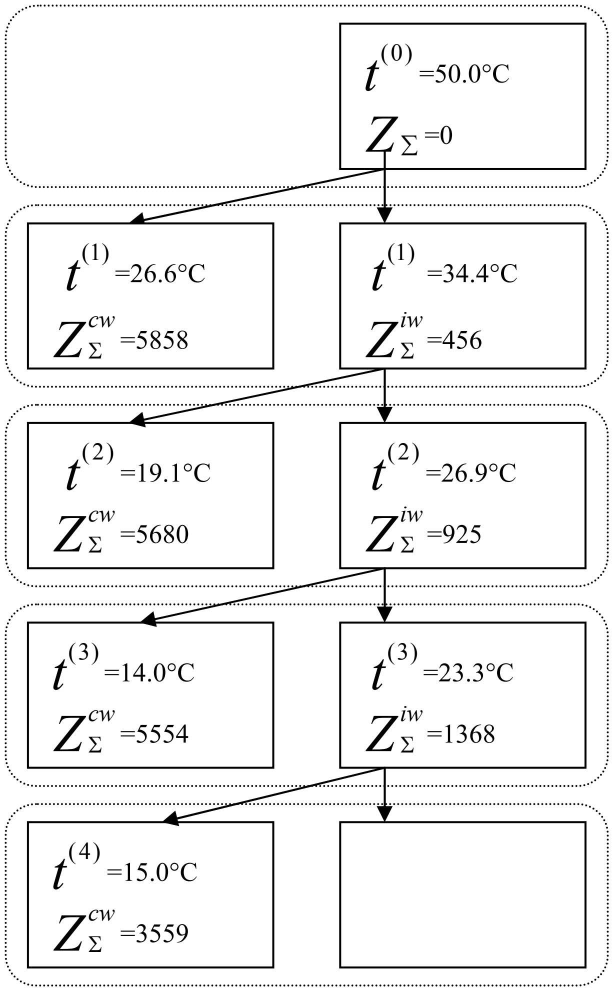

Fig. 4. Tree of possible thermoeconomical expenditure the system of chlorine water refrigeration

IV. Graph of Thermoeconomical Expenditures for Heat Exchanger Network

The more typical case of systems with pair interplay of flow are the heat exchange network (HEN).

The graph of thermoeconomical expenditure for heat exchange network (HEN) with an arbitrary structure is a bipartite graph $Z = (C \cup H, \Gamma_n) = (C \cup H, D)$. The graph consists of the set of nodes $C \cup H$ corresponding to hot (heating) $H = \{h_1, h_2, ..., h_j, ..., h_m\}$ and cold (heated) $C = \{c_1, c_2, ..., c_i, ..., c_n\}$ flows, as well as the set of arcs $D = \{h_i, c_i\}$, i=1,2..., m; j=1,2..., n which represent the possible distribution of thermoeconomical expenditure in the elements of the HEN.

The graph of thermoeconomical expenditure is the simple graph of a view:

$$H \cap C = \varnothing \tag{13}$$

$$(\forall h_i \in H)\Gamma_n h_i \in C \tag{14}$$

$$(\forall c_i \in C)\Gamma_n c_j = \varnothing \tag{15}$$

The equations (13-15) can be proved as follows. Assume, that Eq. (13) is wrong and $H \cap C \neq \varnothing$ or $H \cup C = H_C$. Then, $\exists H_C (\exists! H_C)$ $H_C \subset H$, $H_C \subset C$ and therefore a simultaneous heating and cooling of flows (H_C) will be possible which is dispossessed of physical sense.

The obtained contradiction proves the validity of Eq. (13).

Now, assume, that Eq. (14) is wrong and $(\exists h_i \in H)\Gamma_n h_i \notin C$. Then two versions are possible:

1. $(\exists h_i \in H)\Gamma_n h_i \in H$, resulting in the necessity that $H \cap C \neq \varnothing$, which was demonstrated above as impossible.
2. $(\exists h_i \in H)\Gamma_n h_i = \varnothing$, i.e. the flow h_i is not heating $h_i \notin H$.

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The obtained contradictions demonstrate the impossibility of this allowance so Eq. (14) is proved correct. Eq. (15) can be demonstrated similarly.

Main Properties of Thermoeconomical Expenditure Graph.

The main properties and additional definitions of the thermoeconomical expenditure graph will be illustrated using an example graph shown in Fig. 5(a). This graph demonstrates a possible association of six hot flows $H=\{h_1,h_2,...,h_j,...,h_6\}$ with five cold flows $C=\{c_1,c_2,...,c_i,...,c_5\}$.

The matrix of adjacency of this graph is shown in Fig. 5(b). Like the exergy flow graph [6], the thermoeconomical expenditure graph is oriented, anti-symmetric and not strong tied graph:

$$(\forall h_i \in H)\Gamma_n h_i \neq \varnothing,\ (\forall c_i \in C)\Gamma_n^{-1} c_j \neq \varnothing \qquad (16)$$

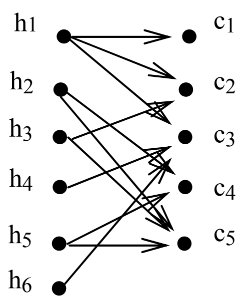
In order to optimize an HEN system, the authors introduce an algorithm based on finding a minimum bearing of the thermoeconomical expenditure graph.

For clarifying this approach a number of additional definitions must be considered.

Cover of the Graph of Thermoeconomical Expenditure

$Z = (C \cup H, D)$ is such subset of arcs $\bar{D} \subset D$ for which one:

$$\forall h_i \in H, \exists (h_i,c_j) \in \bar{D} | (h_i,c_j) \subset D^{+} h_i$$
$$\forall c_j \in C, \exists (h_i,c_j) \in \bar{D} | (h_i,c_j) \subset D^{-} c_j \qquad (17)$$



	c_1	c_2	c_3	c_4	c_5
h_1	1	1	1	0	0
h_2	0	0	0	1	1
h_3	0	1	0	0	1
h_4	0	0	1	0	0
h_5	0	0	0	1	1
h_6	0	0	1	0	0

(a) (b)

Figs. 5. Graph of thermoeconomical expenditure and its matrix of adjacency

An indispensable and sufficient condition of existence of cover of the thermoeconomical expenditure graph is condition (16) and therefore the graph of thermoeconomical expenditure always has cover. This means, that for every cold and hot flow can be find a pair for heat exchange. For example, the set of arcs $D=\{(h_1,c_1), (h_1,c_2), (h_2,c_4),(h_3,c_2), (h_4,c_3), (h_5,c_4) (h_5,c_5), (h_6,c_3)\}$ shown in Fig. 2(a) are one of the possible covers of the thermoeconomical expenditure graph introduced in Fig. 6(a).

In particular, the flow c_3 can be heated by both flows h_4 and h_6. Outgoing from a Boolean –matrix representation of the thermoeconomical expenditure graph, the cover can be defined as a set of units, in which each row and column include at least one unit. In Fig. 6(b) the matrix of the cover shown in Fig. 6(a) is given.

The cover $\bar{D}_0 \subset D$ is minimum, if:

$$\forall\ \bar{D} \subset D, |\ \bar{D}_0 | \leq |\ \bar{D} | \qquad (18)$$

Generally:

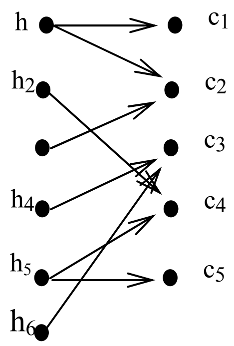
$$| H | \leq | D_0 |$$
$$| C | \leq | D_0 | \qquad (19)$$

With reference to the HEN, Eq. (19) indicates that the minimum number of heat exchanges in the synthesized HEN , can not be less than the number of $|C|$ or $|H|$.

This assumes that within the heat exchanger, each cold and hot flow form only one pair.

The minimum cover $\bar{D}_0 = \{(h_1, c_1), (h_2, c_5) (h_3, c_2) (h_4, c_3) (h_5, c_4), (h_6, c_3)\}$ of the graph of thermoeconomical expenditure shown in Fig. 6(a) is included as Fig. 7(a).

In a Boolean –matrix representation, the minimum cover represents such set of units, that any line and any column of a matrix contains at at least one member from this set, and the total number of members of this set is minimum.



	c_1	c_2	c_3	c_4	c_5
h_1	1	1	0	0	0
h_2	0	0	0	1	0
h_3	0	1	0	0	0
h_4	0	0	1	0	0
h_5	0	0	0	1	1
h_6	0	0	1	0	0

(a) (b)

Figs. 6. Cover of the graph in Fig.1 and its matrix of adjacency

A Boolean –matrix representation of the minimum cover of the graph in Fig. 6(a) is shown in Fig. 7(a). In this case $|\ \bar{D}_0 | = | H | = 6$, corresponding with Eq. (19) which is a consequent of conditions (16) and (17).

The matching of the graph of thermoeconomical expenditure $Z = (C \cup H, \Gamma_n) = (C \cup H, D)$ is mapping Γ_v with number of arcs $V \subset D$ such, that:

$$(\forall h_i \in H^*)\ \Gamma_v h_i \subset \Gamma_n h_i \qquad (20)$$

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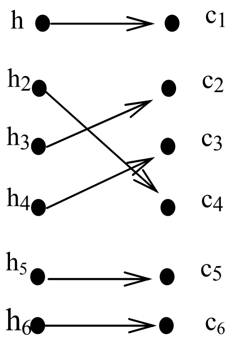
or $(\forall c_j \in C^*)\ \Gamma^{-1}{}_v c_j \subset \Gamma^{-1}{}_n c_j$
$i=1,2, \dots, m;\ j=1,2, \dots, n;$

where the subsets C* and H* satisfy the condition stated in Eq. (9):

$$C^* \subset C;\ H^* \subset H;\ |C^*| = |H^*| \qquad (21)$$



	c_1	c_2	c_3	c_4	c_5
h_1	1	0	0	0	0
h_2	0	0	0	0	1
h_3	0	1	0	0	0
h_4	0	0	1	0	0
h_5	0	0	0	1	0
h_6	0	0	1	0	0

(a) (b)
Figs. 7. Minimum cover of the graph of thermoeconomical expenditure in a Fig. 6 and its matrix of adjacency

Therefore, the number of hot and cold flows, which are included in the considered matching, should match each other in number.

An indispensable and sufficient condition of the existence of a matching (under the theorem of a Konning-Hall):

$$(\forall H^* \subset H)\ |\Gamma_n H^*| \geq |H^*| \qquad (22)$$

i.e. the number of cold flows should not be less in number than the hot flows for a considered matching.

The matching $V_o \subset D$ is maximum, if:

$$(\forall V \subset D)\ |V_o| \geq |V| \qquad (23)$$

In this matching, the greatest possible number of pairs of hot and cold flows is taken into account. The condition stated in Eq. (10) guarantees the consideration of all of the hot flows in finding the maximum matching. To also guarantee this consideration for all set C of cold flow, the condition stated in Eq. (12) should be satisfied:

$$(\forall C^* \subset C)\ |\Gamma^{-1}{}_n C^*| \geq |C^*| \qquad (24)$$

From Eqs. (22) and (24) it follows that $|C| = |H|$.

However, for real HEN it is possible for $|C| \leq |H|$ as well as $|C| > |H|$. Therefore, in order to find a maximum matching it is often necessary to introduce a dummy set of flows corresponding to $|C| = |H|$.

In the previous considered example, a dummy flow c_6 is introduced as shown in Figs. 8. The maximum matching of the graph in Figs. 5 which is now supplemented by node c_6 is shown in a Fig. 8(a).

The matrix, applicable to this maximum matching is given in Fig. 8(b) where:

$$|V_0| = |C| = |H| = 6$$

Matching $V \subset D$, $C^* \subset C$, $H^* \subset H$ is full, if:

$$\forall (h_i, c_j) \subset (D-V)\ |\ h_i \subset H^*,\ \text{or}\ c_j \subset C^* \qquad (25)$$

Maximum matching is always full and the converse is generally insecure.

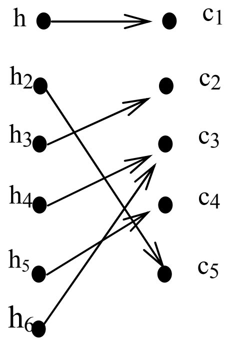
The bearing of the graph of thermoeconomical expenditure $Z = (C \cup H, \Gamma_n)$ is a set $S \subset H \cup C$ for which Eq. (14) holds true:

$$\forall (h_i, c_j) \subset D|\ h_i \subset S,\ \text{or}\ c_j \subset S \qquad (26)$$

Therefore, at least one of nodes of each arc of the graph is included in set S.

The bearing $S_0 \subset S$ of the graph $Z = (C \cup H, D)$ is minimum, if:

$$\forall S \subset H \cup C,\ |S_0| \leq |S| \qquad (27)$$



	c_1	c_2	c_3	c_4	c_5	c_6
h_1	1	0	0	0	0	0
h_2	0	0	0	1	0	0
h_3	0	1	0	0	0	0
h_4	0	0	1	0	0	0
h_5	0	0	0	0	1	0
h_6	0	0	0	0	0	1

(a) (b)
Figs. 8. The maximum matching of the graph of thermoeconomical expenditure in Figs. 5 and its matrix of adjacency

In the synthesizing of HEN, minimum bearing of the graph $Z = (C \cup H, \Gamma_n)$ is a set $S_0 \subset H \cup C$ of hot and cold flows with a minimum total number of members $S_0 = \min |S|$. Number of versions of associations of flows from S_0 among themselves, as well as with the stayed set $(C \cup H) - S_0$ appears maximum.

In addition, according to the Konning theorem

$$|S_0| = |V_0| \qquad (28)$$

Eq. (27) forms the basis for constructing an algorithm for finding a minimum bearing of the graph of thermoeconomical expenditure.

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V. Method of Optimal Synthesis of HEN

Lets assume that the HEN consists of elements from $A = \{a_1, a_2, \ldots, a_k, \ldots, a_l\}$.

Than every a_k, k=1,2,..,l element will be uniquely characterized by an i, j- pair of interacting flows from sets $H = \{h_1, h_2..., h_i,..., h_m\}$ and $C = \{c_1, c_2..., c_j,..., c_n\}$.

The problem of optimal, thermoeconomical synthesizing can be solved by minimizing the sum shown in Eq. (29):

$$Z_{\Sigma}^{min} = min \sum_{i} \sum_{j} Z_{ij} \tag{29}$$

where Z_{ij} –is the thermoeconomical expenditure of interplay of a pair of ij-flows.

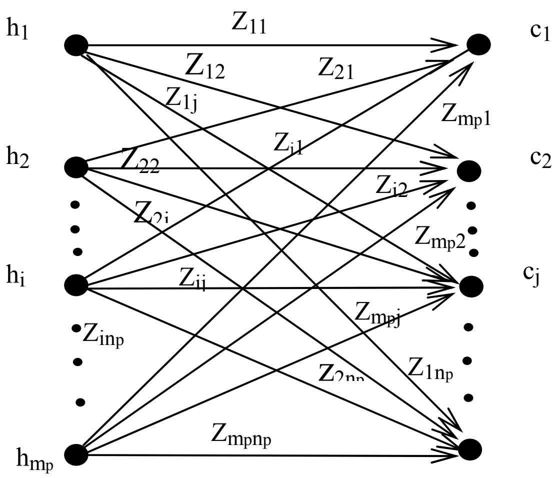

Fig. 9. Graph of thermoeconomical expenditure for a p-step

Lets assume that the optimal system will contain A^{opt} of elements and the procedure of its optimization has s-steps. Also, lets take intermediate p-step ($1 \leq p \leq s$) and assume that on the p-step ($m-m_p$) of interacting flows h_i, ($m > m_p > 0$) and ($n-n_p$) of flows c_j, ($n > n_p > 0$), the desired parameters have been achieved. Then, in the result of minimizing the sum given in Eq. (29) on every p-step, the optimal system with required number of elements A^{opt} will be obtained.

Z_{11}	Z_{12}	…	Z_{1j}	…	Z_{1np}
Z_{21}	Z_{22}	…	Z_{2j}	…	Z_{2np}
…	…		…		…
Z_{i1}	Z_{i2}	…	Z_{ij}	…	Z_{inp}
…	…		…		…
Z_{mp1}	Z_{mp2}	…	Z_{mpj}	…	Z_{mpnp}

Fig. 10. Matrix of graph in Fig. 9

n order to optimize a p-step, the graph of thermoeconomical expenditure for a p-step $\bar{Z} = (\bar{H} \cup \bar{C}, \bar{\Gamma}_n) = (\bar{H} \cup \bar{C}, \bar{D})$ shown in Fig. 9 will be considered.

Here:

$$\begin{gathered} \bar{H} \cap \bar{C} = \varnothing \\ (\forall h_i \in \bar{H})\ \bar{\Gamma}_n h_i \in \bar{C} \\ (\forall c_i \in \ \ \bar{\Gamma}_n c_j = \varnothing) \\ \bar{H} = \{h_1, h_2..., h_i,..., h_{mp}\} \\ \bar{C} = \{c_1, c_2..., c_j,..., c_{np}\} \end{gathered} \tag{30}$$

The set of arcs of the graph $\bar{Z} = (\bar{H} \cup \bar{C}, \bar{D})$ $\bar{D} = \{h_i, c_j\}$, $i=1,2, \ldots, m_p$; $j=1,2, \ldots, n_p$ is defined as:

$$\forall (h_i, c_j) \in \bar{D} \Rightarrow (h_i, c_j) = Z_{ij} \tag{31}$$

Set $\bar{D}$ can be divided into two subsets, $\bar{D}_1 \cup \bar{D}_2 = \bar{D}$ and $\bar{D}_1 \cap \bar{D}_2 = \varnothing$ for which conditions given in Eqs. (32) and (33) are satisfied:

$$\forall (h_i, c_j) \in \bar{D}_1 \Rightarrow Z_{ij} \neq \infty \tag{32}$$

$$\forall (h_i, c_j) \in \bar{D}_2 \Rightarrow Z_{ij} = \infty \tag{33}$$

The version with $D_2 = \varnothing$ is also possible but generally $D_2 \neq \varnothing$.

For finding the distribution of flows that will minimize Eq. (29), a minimum bearing (refer to Eq.(27)) of simple graph $\bar{Z} = (\bar{H} \cup \bar{C}, \bar{D})$ is found which corresponds to a unique matrix of thermoeconomical expenditure, MZ_{ij} of the size $m_p \times n_p$ (shown in Fig. 10). The minimization problem is reduced to finding row vectors ($Z_i = \{Z_{i1}, Z_{i2}, \ldots, Z_{inp}\}$) and vectors of columns ($Z_j = \{Z_{j1}, Z_{j2}, \ldots, Z_{jmp}\}$,) that meet the condition given in Eq. (17).

By transforming the matrix MZ_{ij}, the elements under conditions Eq. (33) are eliminated from consideration.

The outcomes are elements Z_{ij}, which are optimal for the given p-step with a number of elements, A_p.

The algorithm of the solution does not vary as the step is transitioned to p+1 and p = s.

After each step, the sizes of a matrix MZ_{ij} will be diminished and at p = s the matrix becomes equal to zero point.

The number of interacting pairs, obtained on all s-steps gives the thermoeconomical optimal solution with the number of elements stated in Eq. (34):

$$A^{opt} = \sum_{p=1}^{s} A_p \tag{34}$$

VI. Algorithm of P-step Optimization and Numerical Example

In order to find the best value of Z_{Σ}^{min} (refer to Eq.

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(29)) in matrixes MZ_{ij} (shown in Fig. 10) Hungarian method is used [7].

The optimal synthesizing consists of the following main sub-steps:

(I) For i=1,2, …, m_p; j=1,2, …, n_p calculate a value Z_{ij} and to form a matrix MZ_{ij}. The matrix is supplemented to square by (n_p-m_p) zero lines if $m_p < n_p$, or columns if $m_p > n_p$.

(II) Use the Hungarian method [7] to find the elements Z_{ij} of the matrix MZ_{ij} in accordance with the conditions of Eq. (29). Determine the appropriate numbers "i" and "j ".

(III) For all i-flows found in sub-step (II), check the possibility of using it on the (p+1) step. If this is possible, include the i-flow in consideration of (p+1) step. If it is not possible, exclude the i-flow from consideration. The same procedure is made for j-flows from sub-step (II): ensure that these flows achieve the requested parameters.

(IV) If there are j-flows under consideration, go to (p+1) step and continue from sub-step (I). If not, then the optimal synthesis is complete.

$$h_1=\left\{\begin{matrix} Z_{11}=1350, & Z_{12}=1100, & Z_{11},Z_{21} \\ Z_{13}=600, & Z_{14}=1250, & Z_{31},Z_{41} \\ Z_{15}=300, & Z_{16}=1000, & Z_{51},Z_{61} \end{matrix}\right\}=c_1$$

$$h_2=\left\{\begin{matrix} Z_{21}=8000, & Z_{22}=550, & Z_{12},Z_{22} \\ Z_{23}=950, & Z_{24}=250, & Z_{32},Z_{42} \\ Z_{25}=9500, & Z_{26}=200, & Z_{52},Z_{62} \end{matrix}\right\}=c_2$$

$$h_3=\left\{\begin{matrix} Z_{31}=150, & Z_{32}=1150, & Z_{13},Z_{23} \\ Z_{33}=100, & Z_{34}=900, & Z_{33},Z_{43} \\ Z_{35}=700, & Z_{36}=50, & Z_{53},Z_{63} \end{matrix}\right\}=c_3$$

$$h_4=\left\{\begin{matrix} Z_{41}=850, & Z_{42}=7500, & Z_{14},Z_{24} \\ Z_{43}=350, & Z_{44}=1300, & Z_{34},Z_{44} \\ Z_{45}=8500, & Z_{46}=1400, & Z_{54},Z_{64} \end{matrix}\right\}=c_4$$

$$h_5=\left\{\begin{matrix} Z_{51}=1200, & Z_{52}=450, & Z_{15},Z_{25} \\ Z_{53}=1600, & Z_{54}=950, & Z_{35},Z_{45} \\ Z_{55}=1500, & Z_{56}=1650, & Z_{55},Z_{65} \end{matrix}\right\}=c_5$$

$$h_6=\left\{\begin{matrix} Z_{61}=400, & Z_{62}=800, & Z_{16},Z_{26} \\ Z_{63}=1450, & Z_{64}=1700, & Z_{36},Z_{46} \\ Z_{65}=500, & Z_{66}=155, & Z_{56},Z_{66} \end{matrix}\right\}=c_6$$

Fig. 11. The graph of possible thermoeconomical expenditure

As a numerical example, consider a problem of optimal synthesis of a HEN system with one-time interplay of six-elements sets of flows H = {h_1, h_2, h_3, h_4, h_5, h_6} and C = {c_1, c_2, c_3, c_4, c_5, c_6} . The graph of thermoeconomical expenditure (the arcs are not shown), reflecting their possible association, is given in Fig. 7. The application of the described solution algorithm gives the optimal association of flows shown in Fig. 8- as h_1-c_5, h_2-c_4, h_3-c_6, h_4-c_3, h_5-c_2, h_6-c_1. The minimum thermoeconomical expenditure is calculated as Z_Σ^{min} = 300 + 250 + 50 + 350 + 450 + 400 = 1800. The final synthesized optimal HEN is shown in Fig. 13.

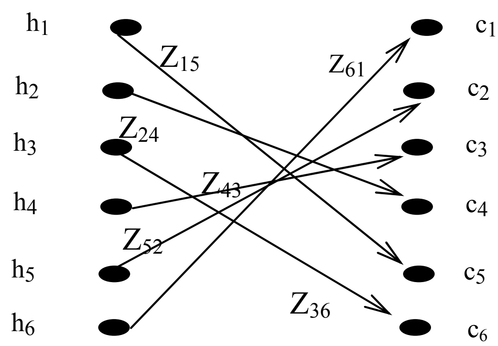

Fig. 12. Optimal matching of graph in Fig.7

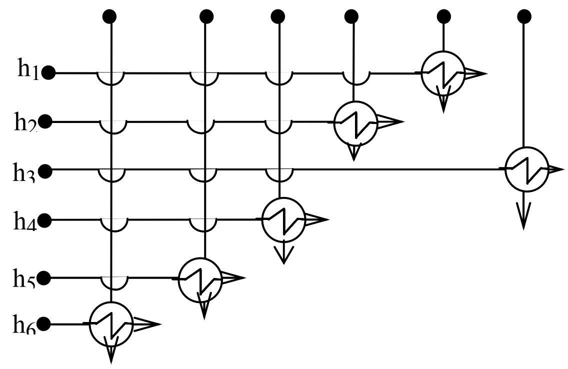

Fig. 13. Optimal system

VII. Conclusion

The problem of optimisation linear systems as well as systems with pair interplay of flows have to be solved separately from the problem of optimisation of systems with arbitrary structure. On the base of these systems features is possible to build the effective procedure of optimisation. The suggested method is based on developing and analyses of appropriate graphs of thermoeconomical expenditure.

For linear systems it allows one to find the optimal variant for homogeneous systems as well as for systems with different types of elements. The method is illustrated by an example of chlorine water refrigeration system optimisation.

For systems with pair interplay of flows method is illustrated with a numerical example of a heat exchanger network with one-time interaction of six flows.

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