Performance Evaluation of Laminar Fully Developed Flow through Ducts with Non-Circular Shapes Subjected to H1 Boundary Condition. Part 2

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Abstract – *Extended performance evaluation criteria (ExPEC) have been used to assess the performance characteristics of single-phase fully developed laminar flow through heat exchangers with rectangular, rectangular with semicircular ends, and elliptical ducts. The heat exchanger with circular tubes has been used as a reference heat transfer unit. The H1 boundary condition has been selected as thermal boundary condition. The performance characteristics of the heat exchangers with non-circular ducts have been evaluated and compared to those of the reference unit for different objectives and constraints imposed. As a common constraint, the cross sectional area of the non-circular duct has been specified. As expected, the analysis of the thermal performance of heat exchangers with non-circular ducts revealed that the selection of the optimal shape of the duct strongly depends on the geometric and thermal-hydraulic constraints imposed on the unit, and the objectives pursued. The use of a general criterion connecting two objectives simultaneously permits to avoid the contradictory results that can be obtained if criteria based on first or second law analysis are implemented alone. The evaluation concerning the superiority of a particular shape of the non-circular duct depends also on the value of the irreversibility distribution ratio, ϕ_o. For the cases FG (fixed geometry criteria) the use of ducts with non-circular shapes is inefficient, whereas in the cases FN (fixed number of tubes) and VG (variable geometry) substantial benefits can be achieved by implementation of rectangular, rectangular ducts with semicircular ends or elliptical ducts if $\phi_o \ll 1$, i.e., the irreversibility due to heat transfer across the wall of the reference channel dominates.* **Copyright © 2013 Praise Worthy Prize S.r.l. - All rights reserved.**

Keywords: *Performance Evaluation Criteria, Single-Phase Laminar Flow, Non-Circular Ducts, Entropy Generation*

Nomenclature

A	Heat transfer surface area [m^2]
c_p	Specific heat capacity [$J\,kg^{-1}K^{-1}$]
D	Reference circular tube diameter [m]
D_h	Hydraulic diameter [m]
h	Heat transfer coefficient [$W\,m^{-2}\,K^1$]
k	Thermal conductivity [$W\,m^{-1}\,K^{-1}$]
L	Tube length [m]
$\dot{m}$	Mass flow rate in tube [$kg\,s^{-1}$]
N_t	Number of tubes
P	Pumping power [W]
p	Wetted perimeter [m]
Δp	Pressure drop [Pa]
$\dot{Q}$	Heat transfer rate [W]
$\dot{S}_{gen}$	Entropy generation rate [$W\,K^{-1}$]

T	Temperature [K]
ΔT	Wall-to-fluid temperature difference [K]
V	Volume of tubes [m^3]
W	Mass flow rate in heat exchanger [$kg\,s^{-1}$]
x	Axial distance along the tube [m]

Greek symbols

ϑ	Temperature difference, $T_w - T$
μ	Dynamic viscosity [Pa s]
ρ	Fluid density [$kg\,m^{-3}$]

Dimensionless groups

A_*	Dimensionless heat transfer surface, $A_w / A_{w,c}$
D_*	Dimensionless tube diameter, D_h / D_c
L_*	Dimensionless tube length, L / L_c
f	Fanning friction factor
f_*	Fanning friction factor ratio, f / f_c

Nu	Nusselt number
Nu_*	Nusselt number ratio, Nu / Nu_c
N_S	Augmentation entropy generation number
N_*	Ratio of number of tubes, $N_t / N_{t,c}$
NTU	Heat transfer units, $4 St\, L / D$
Pr	Prandtl number
P_*	Dimensionless pumping power, P / P_c
Q_*	Dimensionless heat transfer rate, $\dot{Q} / \dot{Q}_c$
Re	Reynolds number
Re_*	Reynolds number ratio, Re / Re_c
St	Stanton number
ΔT_i^*	Dimensionless inlet temperature difference, $\Delta T_i / \Delta T_{i,c}$
V_*	Volume ratio, V / V_c
W_*	Dimensionless mass flow rate, W / W_c
χ	Shape factor, p / D_h
χ_*	Ratio of shape factors, χ / χ_c
ε_*	Ratio of heat exchanger effectiveness, $\varepsilon / \varepsilon_c$
τ	Dimensionless temperature difference, $\Delta T / T$
ϕ_o	Irreversibility distribution ratio

Subscripts

c	Circular tube
f	Fluid
i	Value at $x = 0$
m	Mean
o	Value at $x = L$
w	Wall

I. Introduction

In compact heat exchangers, due to smaller system dimensions, the hydraulic diameter is low, which, in most practical cases, yields essentially laminar flow. It is therefore important to investigate the performance characteristics of different ducted flows, particular in the laminar regime. Because of size and volume constraints in applications for aerospace, nuclear, biomedical engineering and electronics, chemical and process industries, it may be required to use non-circular flow-passage geometries, particularly in compact heat exchangers.

In most of the heat exchangers in service, especially in shell and tube type, circular duct is used. The primary objective of the heat exchanger designer is to use duct geometries that yield: (i) a high value of heat transfer area to volume ratio, (ii) a high value of heat transfer coefficient and (iii) a corresponding value of friction factor. Offering maximum compactness, i.e., highest surface area to volume ratio, however, is not enough for selection of duct geometry.

A designer should also take into consideration the overall thermal hydraulic behavior of the flow through the ducts.

The performance of conventional heat exchangers, for single-phase flows in particular, can be substantially improved by many augmentation techniques, resulting in the design of high-performance thermal design systems. On the basis of the first-law analysis Webb and Bergles [1]-[2] have proposed performance evaluation criteria (PEC) that define the performance benefits of an exchanger having augmented surfaces, relative to standard exchanger with smooth surfaces subject to various objectives and design constraints.

On the other hand, it is well established that the minimization of the entropy generation in any process leads to the conservation of useful energy. A thermodynamic basis to evaluate the merit of augmentation techniques by second-law analysis has been proposed by Bejan [3]-[4] developing the entropy generation minimization (EGM) method. This method has been used as a general criterion for estimating and minimizing the irreversibilities and optimum-design method for heat exchangers. The method has been extended by Zimparov [5]-[6] including the effect of fluid temperature variation along the length of a tubular heat exchanger, and new information has been added assessing two objectives simultaneously. The EGM method combined with the first law analysis provides the most powerful tool for the analysis of the thermal performance of any augmentation technique.

In many instances, the designer is faced with an existing equipment, where the space occupied by the cooling passage is at a premium and the heat and mass flow rates are limited by the size of an existing or retrofit pump or fan. In these situations, where a coolant passage must be designed so that the volume of the passage is restricted to some value and the heat rate and mass flow rate of the coolant are dictated by the available equipment, the designer may ask the question: Is there an optimal cross-sectional area and optimal length for the coolant passage that minimizes entropy generation and allows for the best performance?

A number of studies have been focused on the calculation and minimization of entropy generation in the fundamental fully-developed convective flow configuration in a duct. In most of these studies the entropy generation has been calculated and minimized in ducts with various cross-sectional shapes for laminar and turbulent configurations, with constant heat transfer rate per unit length, with constant heat flux, or with constant wall temperature, also in flows with temperature dependent physical properties [7]-[19].

Circular ducts are generally used in tubular heat exchangers and most of the augmentation techniques have been applied to such tubes. As to the ducts with non-circular cross section the improvement of the performance of one geometry with respect to another

depends on the duct geometry, inlet-to-wall temperature ratio and the operating Reynolds number for a given fluid and a certain duct length. Most of the analyses related to non-circular cross sections (namely, square, equilateral triangle, rectangular, sinusoidal, etc.) and with single-phase laminar flow have been conducted on the basis of the second law analysis.

In a recent paper, Chakraborty and Rey [20] have evaluated the performance of a square duct with rounded corners, for single-phase laminar flow using the PEC criteria identified by Webb and Bergles [1]-[2] taken as objective functions for each case, and second law analysis in an attempt to find out an optimal operating point, i.e., a particular radius of curvature for the corners, which is advantageous from both first and second law analyses.

For any duct with non-circular shape, the size is determined by either the hydraulic diameter D_h, or the cross sectional area A_f, since these parameters are related through the shape factor $\chi = 4A_f / D_h^2$. In this regard, two different common constraints can be imposed – specified cross-sectional area $A_f^* = 1$, or specified hydraulic diameter of the ducts, $D_* = 1$. The results of performance evaluation criteria for the first case, $A_f^* = 1$, and constant wall temperature as a boundary condition, have been published recently by Petkov et al. [22].

The present study aims at to extend the previous study [21] with more non-circular ducts as axial corrugated, rectangular, rectangular with semicircular ends, square with rounded corners, triangular with rounded corners, and elliptical ducts. In Part 1 of this study [23], we presented the results obtained for performance characteristic of axial corrugated, square with rounded corners, and triangular with rounded corners ducts.

In Part 2 of this study, the performance characteristic of rectangular, rectangular with semicircular ends, Fig. 1, and elliptical ducts have been evaluated.

The boundary condition of the wall is H1 boundary condition. The rationale of the study is to evaluate the thermal performance of tubular heat exchangers with non-circular cross section ducts and single-phase laminar flow in the channels. Similar to the studies [21],[22], the common constraint $A_f^* = 1$ is imposed. In this case, the ratio of diameters of the ducts is a consequence, $D_* = \chi_*^{-1/2}$.

The heat exchanger with circular tubes has been used as a reference heat transfer unit. Using the first and second laws simultaneously, the performance characteristics of units with non-circular tubes have been evaluated for different objectives and constraints imposed and compared to those of the reference unit.

II. Equations Based on the Entropy Production Theorem

Consider the energy and entropy balance of the general internal flow configuration, where fluid flows through a duct with a cross sectional area A_f, a perimeter p, and hydraulic diameter $D_h = 4A_f / p$.

The shape of the cross section is arbitrary but constant over the entire length of the duct. The flow is single-phase, fully developed laminar, incompressible and Newtonian.

Since the cross-sectional area for the flow does not change in the axial direction, the mass flow rate ($\dot{m}_t$) through the duct remains constant. For H1 boundary condition (i.e., constant heat flow per unit axial length and uniform temperature along the periphery of the duct at a given axial location), q' is consider to be known. Following the model developed by Zimparov [5], for tubular full-size heat exchangers, the rate of entropy generation can be expressed [23] as:

$$\dot{S}_{gen} = \frac{\dot{Q}^2}{N_t^2 k_f Nu \chi L_t T_i T_o} + \frac{8W^2 (f\,Re)\mu L_t}{N_t^2 \rho^2 \chi D_h^4 T_i} \qquad (1)$$

where: $\dot{Q} = N_t \dot{Q}_t$, $\dot{Q}_t = \dot{m}_t c_p (T_o - T_i)$, $W = \dot{m}_t N_t$, $A = pLN_t$, $T_o = T_i + \frac{hp\Delta T}{\dot{m}_t c_p} L$. (The heated and wetted perimeters are assumed to be the same.)

Following Bejan [3]-[4], the augmentation entropy generation number N_S can be presented as:

$$N_S = \frac{\dot{S}_{gen}}{\dot{S}_{gen,circle}} = \frac{N_{S,T} + \phi_o N_{S,P}}{1 + \phi_o} =$$
$$= \frac{1}{1 + \phi_o} \left(N_{S,T} + \phi_o N_{S,P} \right) \qquad (2)$$

where [5], [23]:

$$N_{S,T} = \frac{Q_*^2}{Nu_* N_*^2 \chi_* L_* T_o^*} \qquad (3a)$$

$$T_o^* = \left[\frac{T_{i,c}}{T_{o,c}} + \frac{Q_*}{W_*} \left(1 - \frac{T_{i,c}}{T_{o,c}} \right) \right] \qquad (3b)$$

Fig. 1. The geometrical parameters of square duct with rounded corners

$$N_{S,P} = \frac{W_*^2 L_* (f\,Re)_*}{N_*^2 \chi_* D_*^4} \equiv P_* \qquad (3c)$$

$$\phi_o = \frac{8 W_c^2 \mu L_c^2 (f\,Re)_c}{\rho^2 D_c^4} \times \frac{k_f Nu_c T_{o,c}}{\dot{Q}_c^2} \qquad (3d)$$

The numerical value of the irreversibility distribution ratio $\phi_o = \left(\dot{S}_{gen,\Delta P} / \dot{S}_{gen,\Delta T}\right)_c$, describes the thermodynamic mode in which the circular tube passage is meant to operate. This mode can be expressed also by the value of Bejan number, $Be = \left(\dot{S}_{gen,\Delta T} / \dot{S}_{gen}\right)_c$ or $Be = 1/(1 + \phi_o)$. The PEC as suggested by Webb and Bergles [1], [2] characterize nearly all the PEC and some of them will be considered below. The equations are developed for ducts with different cross sectional shape and the equations (in dimensionless form) for inside single-phase laminar flow are:

$$A_* = p_* L_* N_* = \chi_* D_* N_* L_* \qquad (4)$$

$$P_* = \frac{W_*^2 L_* (f\,Re)_*}{N_*^2 \chi_* D_*^4} \qquad (5)$$

$$Q_* = W_* \varepsilon_* \Delta T_i^* \qquad (6)$$

$$W_* = Re_* \, p_* N_* = Re_* \, \chi_* D_* N_* \qquad (7)$$

where:

$$\chi_* = p_* / D_*$$

III. Performance Evaluation Criteria

The performance evaluation criteria, as suggested by Webb and Bergles [1]-[2] and extended by Zimparov [5] have been considered in this study. These criteria are based on the use of first and second law analyses in the pursuit of two objectives simultaneously. In this study the geometrical and regime parameters of the reference channel (smooth circular tube) are selected to fulfill the requirement of $4 L_c / (D_c \, Re \, Pr) = 1$.

The performance characteristics of heat exchangers with non-circular tubes, such as rectangular, rectangular with semicircular ends, and elliptical shapes are compared to those of the reference circular tubes heat exchanger. The values of the shape factor χ, friction factor f, and Nusselt number Nu of non-circular channels are taken from and Shah and London [24].

While obtaining the augmentation entropy generation number, the irreversibility distribution ratio for the circular configuration, ϕ_o, varies in the range $10^{-3} \le \phi_o \le 10^3$ ($0.001 < Be < 0.999$).

III.1. Fixed Geometry Criteria (FG)

These criteria involve a replacement of circular tubes by tubes with non-circular shape of equal length and cross sectional area. The FG-1 cases seek increased heat duty for constant exchanger flow rate and heat transfer area.

The FG-2 criteria have the same objective as FG-1, but requires that the non-circular tube design to operate at the same pumping power as the reference circular tube design. The third criterion, FG-3, attempts to effect reduced pumping power for constant heat duty and surface area.

III.1.1. Case FG-1a

The objective functions of the case FG-1a are increased heat rate $Q_* > 1$, decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them as a general performance criterion $N_S / Q_* < 1$.

The constraints imposed are: $W_* = 1$, $\Delta T_i^* = 1$, $A_* = 1$, $L_* = 1$, $A_f^* = 1$. The consequences of these constraints are $D_* = \chi_*^{-1/2}$, $Re_* = 1$, $P_* > 1$, $N_* < 1$, $V_* < 1$.

The constraint of equal heat transfer surface area, $A_* = 1$, requires $N_* = \chi_*^{-1/2}$. In this case the total bundle cross-sectional area and the volume ratio are $A_{f,tot}^* = N_* A_f^* = \chi_*^{-1/2}$, and $V_* = L_* A_{f,tot}^* = \chi_*^{-1/2}$.

The constraint of equal mass flow $W_* = 1$, requires $Re_* = 1$. The Eqs. (5) and (6) yield:

$$P_* = \chi_*^2 (f\,Re)_* \qquad (8)$$

and:

$$Q_* = \varepsilon_* \qquad (9)$$

where:

$$\varepsilon_* = 1.229 \frac{Nu}{1 + Nu} \qquad (10)$$

The Eqs. (3a)-(3c) yield:

$$T_o^* = 0.981 + 0.019 \varepsilon_* \qquad (11)$$

$$N_{S,T} = \frac{Q_*^2}{Nu_* T_o^*} \qquad (12)$$

$$N_{S,P} \equiv P_* = \chi_*^2 (f\,Re)_* \qquad (13)$$

and the augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o}\left(\frac{Q_*^2}{Nu_* T_o^*} + \phi_o \chi_*^2 \left(f\,Re\right)_*\right) \qquad (14)$$

The values of Q_* are calculated by Eq. (9), and the variation of Q_* with χ_* are shown in Fig. 2(a). The value of Q_* for heat exchangers with elliptical ducts (for all values of χ_*), rectangular ducts (for $\chi_* > 1.56$), and rectangular ducts with semicircular ends (for $\chi_* > 1.31$) the goal $Q_* > 1$ can be achieved. The benefit however is not much: less than 3% for elliptical ducts, Fig. 2(d), almost 7% for rectangular ducts with semicircular ends
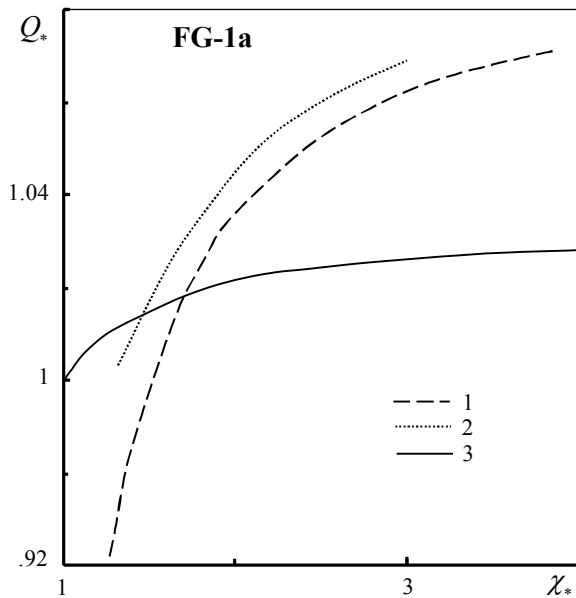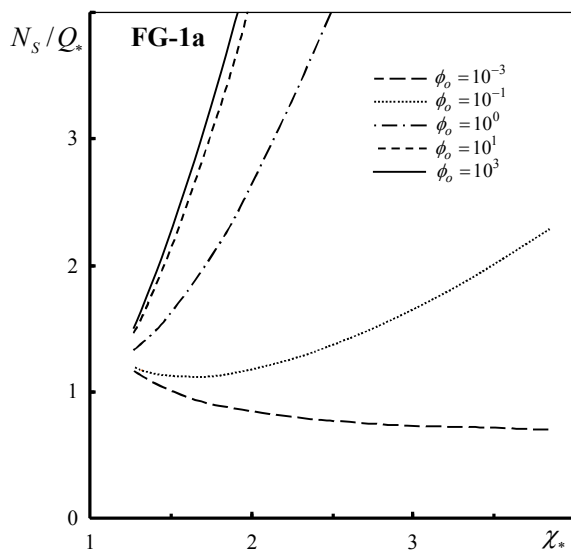
($\chi_* = 3.0$), Fig. 2(c), and can reach an increase of 9% for rectangular ducts, but for the value of $\chi_* \approx 16.6$, Fig. 2(b).

The implementation of first and second law analysis simultaneously, as a general criterion N_S / Q_*, Figs. 2(b)-2(d), shows that the use of ducts with non-circular shapes as studied can bring about some benefit only for very small values of ϕ_o, $\phi_o \ll 10^{-3}$ ($Be > 0.99$).

The design with rectangular ducts with semicircular ends seems to be the most attractive and the benefit increases with the increase of χ_*.
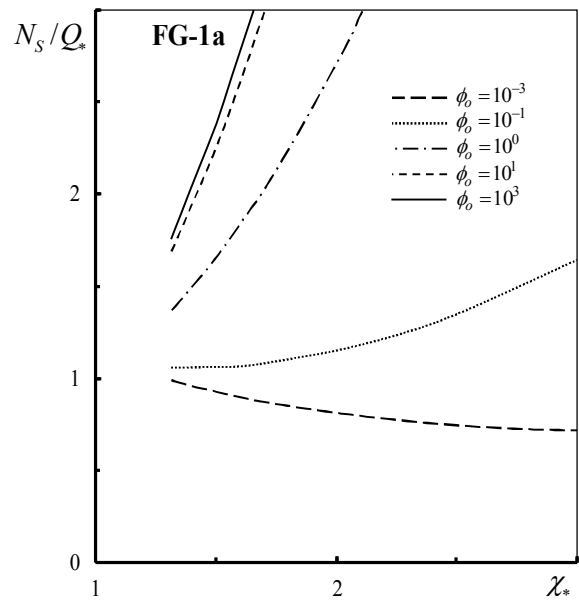
Fig. 2(a). The variation of Q_* with χ_* (1- rectangular ducts, 2- rectangular ducts with semicircular ends, 3-elliptical ducts)

Fig. 2(c). The variation of N_S / Q_* with χ_* (rectangular ducts with semicircular ends)

Fig. 2(b). The variation of N_S / Q_* with χ_* (rectangular ducts)
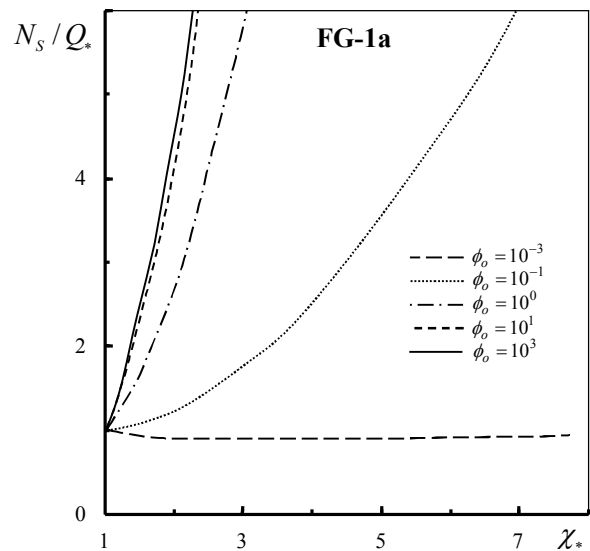
Fig. 2(d). The variation of N_S / Q_* with χ_* (elliptical ducts)

III.1.2. Case FG-2a

The objective functions of the case FG-2a are increased heat duty $Q_* > 1$, decreased entropy generation numbers $N_S < 1$, and simultaneous effect of the both of them $N_S / Q_* < 1$. The constraints imposed are: $\Delta T_i^* = 1$, $P_* = 1$, $A_* = 1$, $L_* = 1$, $A_f^* = 1$. The consequences of these constraints are $D_* = \chi_*^{-1/2}$, $W_* < 1$, $N_* < 1$. The constraint $A_* = 1$ requires $N_* = \chi_*^{-1/2}$, $A_{f,tot}^* = \chi_*^{-1/2}$, and $V_* = \chi_*^{-1/2}$.

The constraint $P_* = 1$, requires:

$$W_* = Re_* = \chi_*^{-1}\left(f\,Re\right)_*^{-1/2} \qquad (15)$$

In this regard, Eqs. (2)-(7) yield:

$$Q_* = \frac{\varepsilon_*}{\chi_*\left(f\,Re\right)_*^{1/2}} \qquad (16)$$

$$N_{S,T} = \frac{Q_*^2}{Nu_* T_o^*} \qquad (17)$$

where ε_* and T_o^* are calculated by Eqs. (10) and (11).

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o}\left(\frac{Q_*^2}{Nu_* T_o^*}+\phi_o\right) \qquad (18)$$

The values of Q_* calculated by Eq. (16), are shown in Fig. 3(a). As seen, for all non-circular ducts studied $Q_* < 1$ and it decreases very fast with the increase of χ_*. However, when the two objectives are pursued simultaneously and general criterion N_S / Q_* is implemented in some cases benefit can be achieved, Figs. 3(b)-3(d). For all non-circular ducts studied the benefit can be achieved for small values of χ_*, $\phi_o < 1$, and the with the decrease of ϕ_o ($\phi_o \ll 1$). It should be pointed out that $Q_* < 1$, and the benefit obtained of low values of N_S / Q_* should be restricted to small values of χ_* to sustain the value of Q_* as higher as possible.

III.1.3. Case FG-3

The objective functions of the case FG-3 are lower pumping power $P_* < 1$, decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S P_* < 1$.
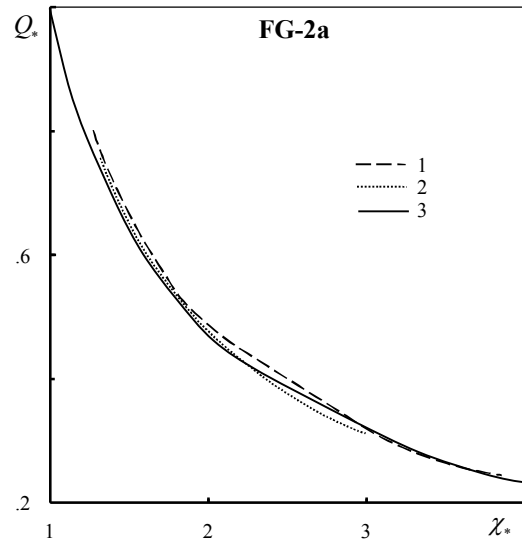

Fig. 3(a). The variation of Q_* with χ_* (1- rectangular ducts, 2- rectangular ducts with semicircular ends, 3- elliptical ducts)
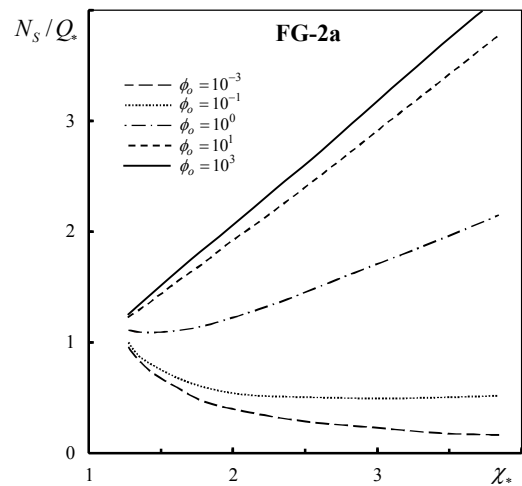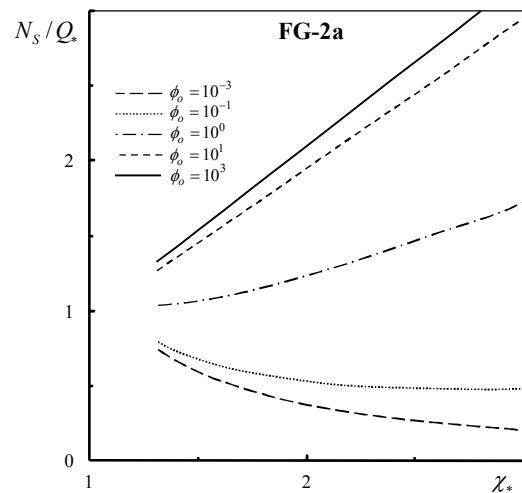

Fig. 3(b). The variation of N_S / Q_* with χ_* (rectangular ducts)

Fig. 3(c). The variation of N_S / Q_* with χ_* (rectangular ducts with semicircular ends)
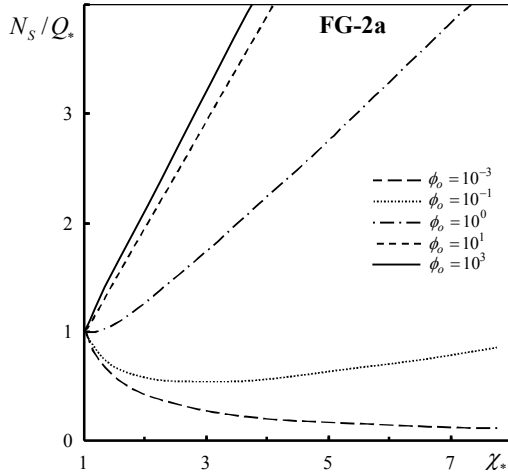
Fig. 3(d). The variation of N_S / Q_* with χ_* (elliptical ducts)

Fig. 4(a). The variation of P_* with χ_* (1- rectangular ducts, 2- rectangular ducts with semicircular ends, 3- elliptical ducts)

The constraints imposed are: $Q_* = 1$, $\Delta T_i^* = 1$, $A_* = 1$, $L_* = 1$, $A_f^* = 1$. The consequences of these constraints are $D_* = \chi_*^{-1/2}$, $A_{f,tot}^* = V_* = \chi_*^{-1/2}$, and $N_* < 1$.

The Eqs. (2)-(7) yield $N_* = \chi_*^{-1}$, $W_* = Re_* = \varepsilon_*^{-1}$, where ε_* is calculated by Eq. (10):

$$P_* = \frac{\chi_*^2 \left(f\, Re \right)_*}{\varepsilon_*^2} \qquad (19)$$

$T_o^* = 0.981 + 0.019\varepsilon_*$, $N_{S,P} = P_*$, and:

$$N_{S,T} = \frac{1}{Nu_* T_o^*} \qquad (20)$$

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o} \left(\frac{1}{Nu_* T_o^*} + \phi_o \frac{\chi_*^2 \left(f\, Re \right)_*}{\varepsilon_*^2} \right) \qquad (21)$$

The calculated values of P_* by Eq. (19) are presented in Fig. 4(a), whereas the results of using the general criterion $N_S P_*$ can be seen in the Figs. 4(b)-4(d). It is obviously that in this case, the heat transfer performance of the heat exchanger cannot be improved by the use of non-circular ducts irrespective the values of ϕ_o and χ_*.

The advantage of implementation of circular tubes in the design of heat exchanger is unimpeachable.

III.2. Fixed Number of Tubes Criteria (FN)

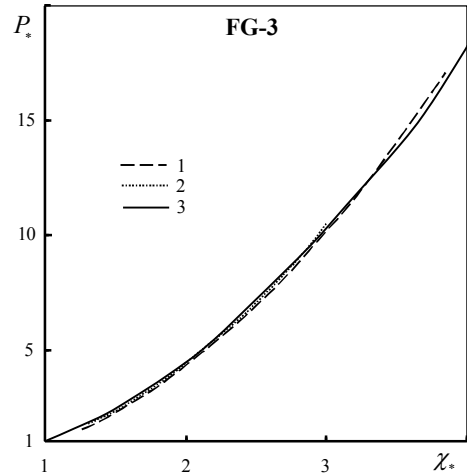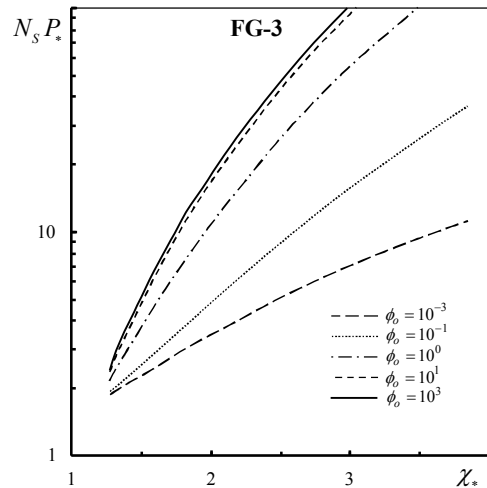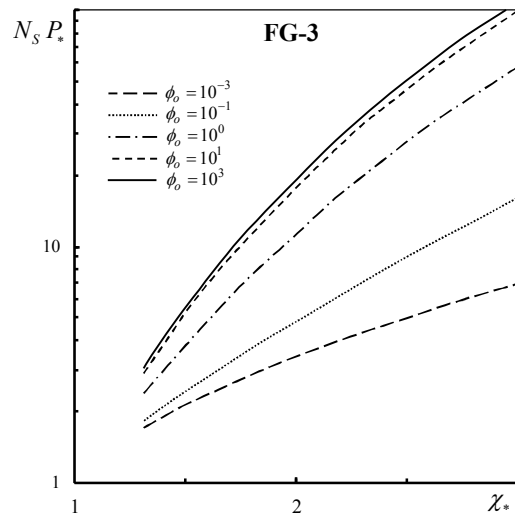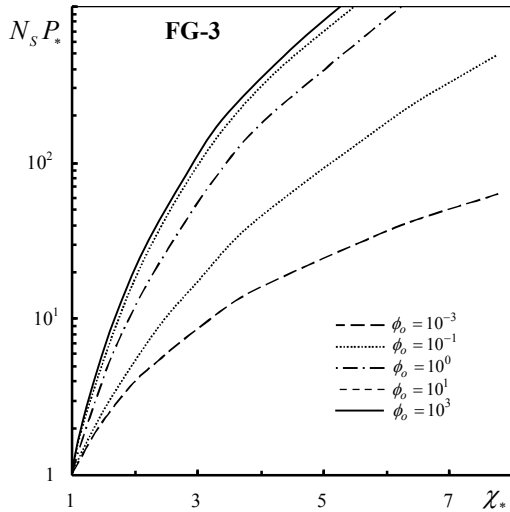These criteria maintain constant number of tubes.

Fig. 4(b). The variation of $N_S P_*$ with χ_* (rectangular ducts)

Fig. 4(c). The variation of N_S / Q_* with χ_* (rectangular ducts with semicircular ends)

Fig. 4(d). The variation of $N_S P_*$ with χ_* (elliptical ducts)

The objective of FN-1 is reduced surface area, via reduced tubing length, for constant pumping power.

Reduced flow rate will probably be required to satisfy the constant pumping power criterion. The objective of FN-2 is reduced pumping power.

III.2.1. Case FN-1

The objective functions of the case FN-1 are reduced heat transfer area $A_* < 1$ ($L_* < 1$), decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S A_* < 1$.

The constraints imposed are: $N_* = 1$, $Q_* = 1$, $P_* = 1$, $\Delta T_i^* = 1$, and $A_f^* = 1$. The consequences of the constraints are $D_* = \chi_*^{-1/2}$, $L_* < 1$, $A_{f,tot}^* < 1$, $V_* < 1$.

The Eqs. (2)-(7) yield:

$$W_* = \chi_* Re_* = \varepsilon_*^{-1} \qquad (22)$$

$$L_* = \frac{\varepsilon_*^2}{\chi_* (f Re)_*} \qquad (23)$$

$$A_* = N_* \chi_* L_* D_* = \frac{\varepsilon_*^2}{\chi_*^{1/2} (f Re)_*} \qquad (24)$$

where ε_* is calculated by Eq. (10), $N_{S,P} = P_* = 1$, $T_o^* = 0.981 + 0.019 \varepsilon_*$:

$$N_{S,T} = \frac{(f Re)_*}{Nu_* T_o^* \varepsilon_*^2} \qquad (25)$$

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o} \left(\frac{(f Re)_*}{Nu_* T_o^* \varepsilon_*^2} + \phi_o \right) \qquad (26)$$

The calculated values of A_* by Eq. (24) are shown in Fig. 5(a).

The variations of A_* with χ_* clearly show that the use of non-circular ducts as studied can substantially diminish the needed heat transfer area.

The implementation of the general criterion $N_S A_*$ also confirms the optimistic expectations, Figs. 5(b)-5(d). If the heat exchanger is designed with rectangular ducts, Fig. 5(b), the general criterion $N_S A_*$ does not depend on ϕ_o for $\chi_* \approx 1.65$, and $N_S A_* \approx 0.74$.

For $\chi_* < 1.65$, the greatest benefit can be achieved when $\phi_o \gg 1$ ($\phi_o = 10^3$), whereas when $\chi_* > 1.65$, the best performance of the unit can be achieved for values $\phi_o \ll 1$ ($\phi_o = 10^{-3}$).

Nevertheless, when $\chi_* > 1.65$, irrespective the value of ϕ_o, a great benefit can be obtained and it increases with the increase of χ_*.

The use of rectangular ducts with semicircular ends, is also beneficial, even though a little more beneficial than the ordinary rectangular ducts, Fig. 5(c). The benefit increases with the decrease of ϕ_o and the greatest benefit is available at $\phi_o \ll 1$ ($\phi_o = 10^{-3}$).

The performance characteristic of the unit with elliptical ducts, Fig. 5(d), is quite interesting. The benefit increases with the increase of χ_* and it does not depend on the value of ϕ_o.
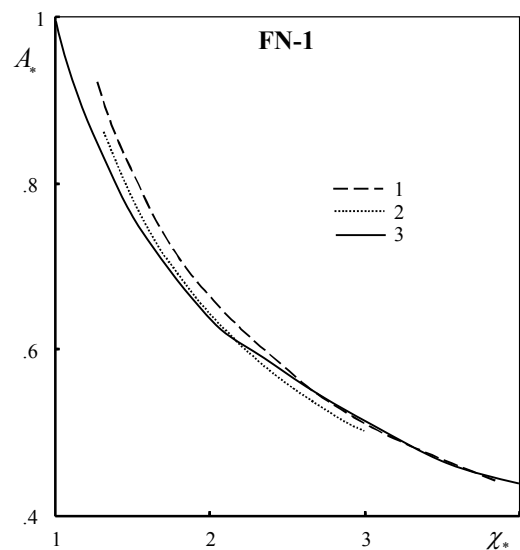
Fig. 5(a). The variation of P_* with χ_* (1- rectangular ducts, 2- rectangular ducts with semicircular ends, 3-elliptical ducts)
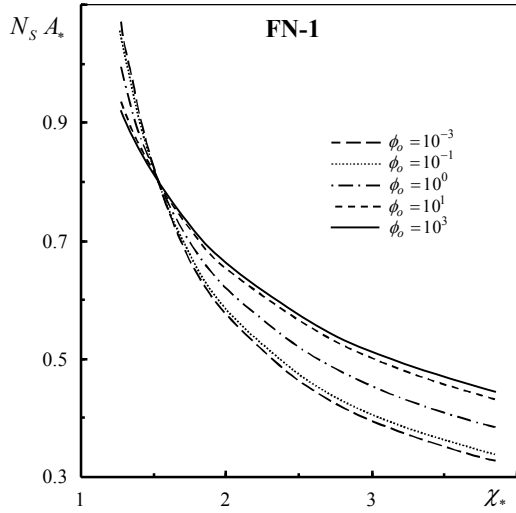
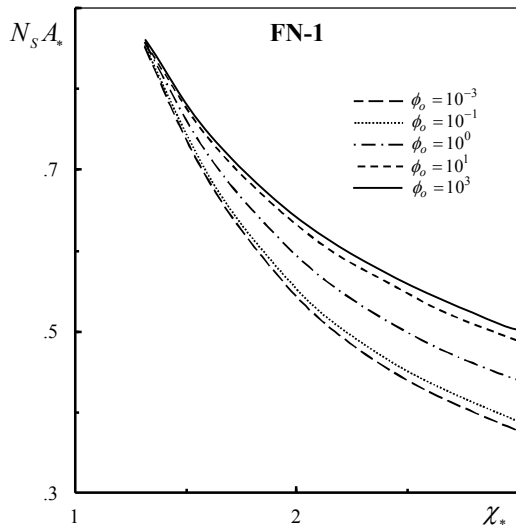Fig. 5(b). The variation of $N_S A_*$ with χ_* (rectangular ducts)

Fig. 5(c). The variation of $N_S A_*$ with χ_*
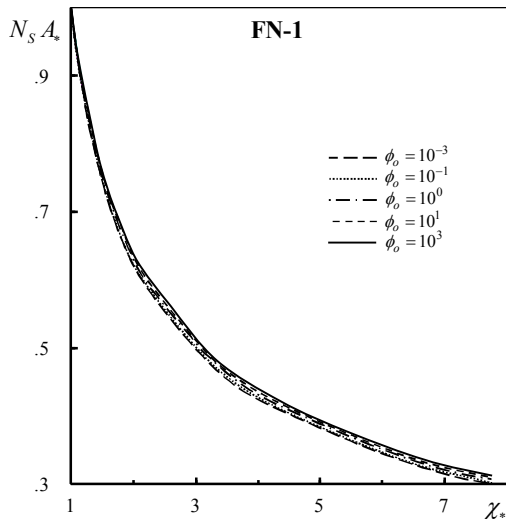(rectangular ducts with semicircular ends)

Fig. 5(d). The variation of $N_S A_*$ with χ_* (elliptical ducts)

In the case FN-1, the units with rectangular, rectangular ducts with semicircular ends or elliptical ducts possess almost one and the same performance characteristics and any one of them can be used as a better choice compare to the circular tube.

III.2.2. Case FN-2

The objective functions of the case FN-2 are lower pumping power $P_* < 1$, decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S P_* < 1$. The constraints imposed are: $Q_* = 1$, $A_* = 1$, $\Delta T_i^* = 1$, $N_* = 1$, $A_f^* = 1$. The consequences of the constraints are $D_* = \chi_*^{-1/2}$, $L_* < 1$, $A_{f,tot}^* < 1$, $V_* < 1$. Eqs. (2)-(7) yield $L_* = V_* = \chi_*^{-1/2}$, $W_* = \varepsilon^{-1}$, $Re_* = \left(\chi_*^{1/2} \varepsilon_* \right)^{-1}$:

$$P_* = \frac{\chi_*^{1/2} \left(f\,Re \right)_*}{\varepsilon_*^2} \tag{27}$$

$T_o^* = 0.981 + 0.019 \varepsilon_*$, where ε_* is calculated by Eq. (10). $N_{S,P} = P_*$, whereas:

$$N_{S,T} = \frac{1}{Nu_* \chi_*^{1/2} T_o^*} \tag{28}$$

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o} \left[\frac{1}{Nu_* \chi_*^{1/2} T_o^*} + \phi_o \frac{\chi_*^{1/2} \left(f\,Re \right)_*}{\varepsilon_*^2} \right] \tag{29}$$

The calculated values of P_* by Eq. (27) are presented in Fig. 6(a). As seen, the variation of P_* with χ_* revealed that the first objective $P_* < 1$ cannot be achieved by the use of the non-circular ducts as studied.

On the other hand, however, the these non-circular ducts can substantially decrease the generated entropy of the unit and the implementation of the general criterion $N_S P_* < 1$ shows that benefit can be obtained, particularly for $\phi_o \leq 1$. For instance, the use of rectangular or rectangular ducts with semicircular ends can bring about benefit for $\phi_o \leq 10^{-1}$, Figs. 6(b)-6(c).

For $\chi_* \approx 3.0$, the benefit is almost one and the same, $N_S P_* \approx 0.178$. The variation of $N_S P_*$ with χ_* for elliptical ducts, Fig. 6(c), has the same tendency as the unit with square ducts with rounded corners [23] and the results are almost the same.
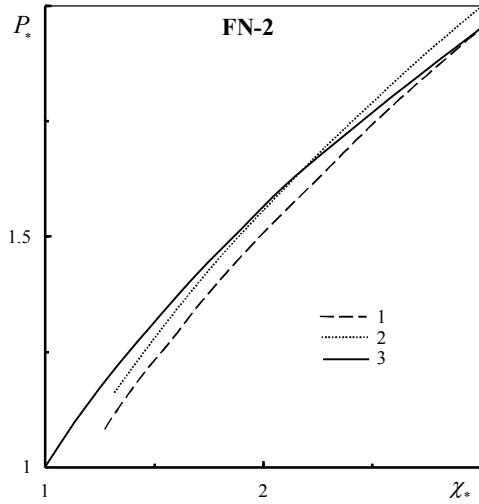
Fig. 6(a). The variation of P_* with χ_* (1- rectangular ducts, 2- rectangular ducts with semicircular ends, 3- elliptical ducts)
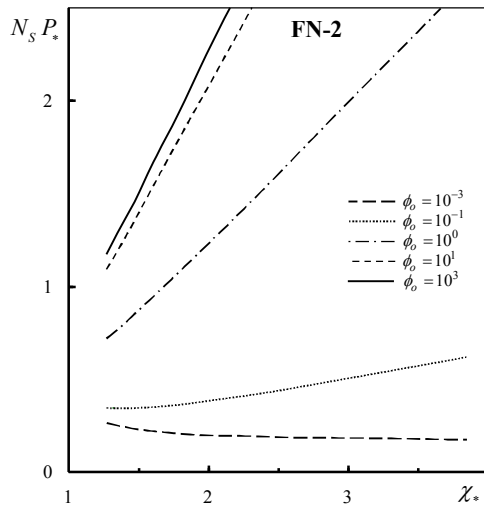

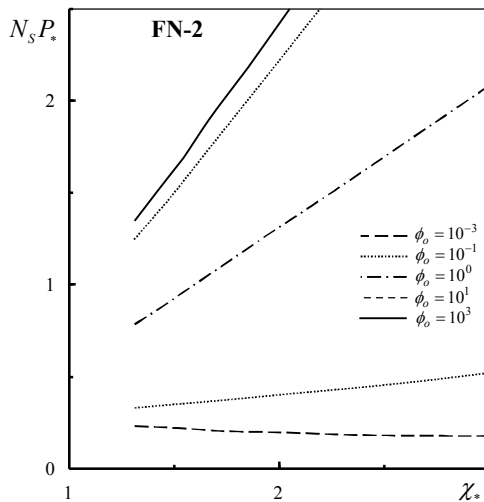Fig. 6(b). The variation of $N_S P_*$ with χ_* (rectangular ducts)

Fig. 6(c). The variation of $N_S P_*$ with χ_*
(rectangular ducts with semicircular ends)

Consequently, for the case FN-2 the use of non-circular ducts can bring about some benefit only if $\phi_o \ll 1$.

III.3. Variable Geometry Criteria (VG)

The criteria VG are applicable when the heat exchanger is "sized" for a required thermal duty with specified flow rate.

III.3.1. Case VG-1

The objective functions of the case VG-1 are lower heat transfer area $A_* < 1$, decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S A_* < 1$. The constraints imposed are: $Q_* = 1$, $W_* = 1$, $P_* = 1$, $\Delta T_i^* = 1$, $A_f^* = 1$. The consequences of the constraints are $D_* = \chi_*^{-1/2}$, $L_* < 1$, $A_{f,tot}^* > 1$, $V_* < 1$. The Eqs. (2)-(7) yield: $\varepsilon_* = 1$, $T_o^* = 1$, $N_{S,P} = P_* = 1$:

$$A_* = Nu_*^{-1} \tag{30}$$

(since $Q_* = 1$, and $\Delta T_m^* = 1$):

$$N_* = \left[\frac{\chi^{1/2} (fRe)_*}{Nu_*} \right]^{1/3} \tag{31}$$

$$L_* = \left[\chi_*^2 (fRe)_* Nu_*^2 \right]^{-1/3} \tag{32}$$

$$Re_* = \chi_*^{1/6} (fRe)_*^{1/3} Nu_*^{2/3} \tag{33}$$

$$N_{S,T} = \left[\frac{Nu_*}{\chi_*^2 (fRe)_*} \right]^{1/3} \tag{34}$$

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o} \left\{ \frac{Nu_*^{1/3}}{\chi_*^{2/3} (fRe)_*^{1/3}} + \phi_o \right\} \tag{35}$$

The calculated values of A_* by Eq. (30) are presented in Fig. 7(a) as the variation of A_* with χ_*.

As seen, the use of rectangular ducts with semicircular ends or elliptical ducts always improves the performance characteristic, $A_* < 1$, despite the value of χ_*.

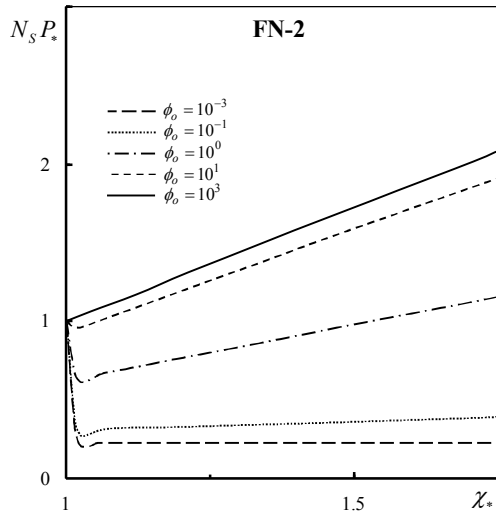The benefit can also be obtained with rectangular ducts if $\chi_* > 1.7$.

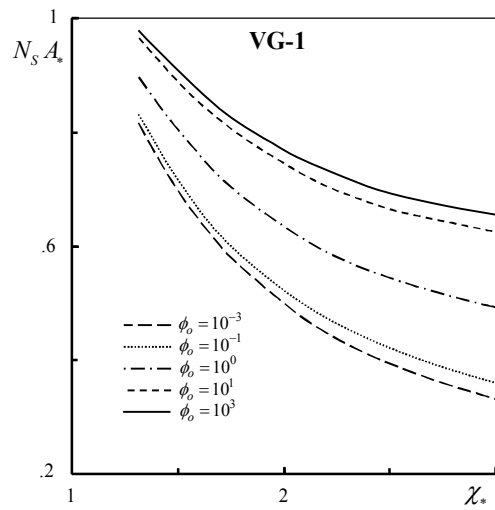Fig. 6(d). The variation of $N_S P_*$ with χ_* (elliptical ducts)

Fig. 7(c). The variation of $N_S A_*$ with χ_*
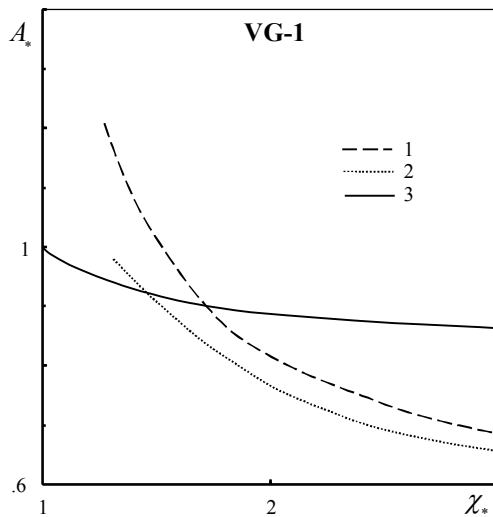(rectangular ducts with semicircular ends)

Fig. 7(a). The variation of A_* with χ_* (1- rectangular ducts, 2-
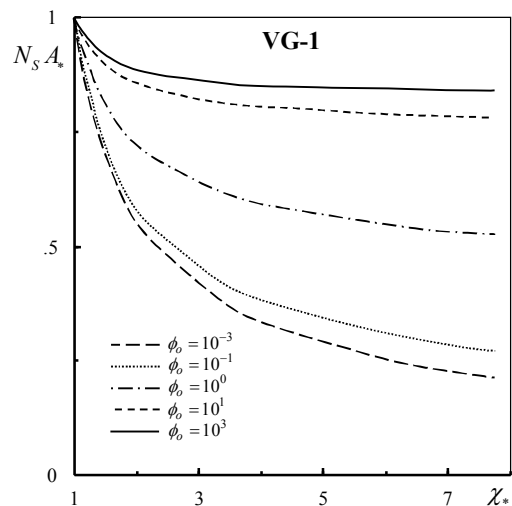rectangular ducts with semicircular ends, 3- elliptical ducts)

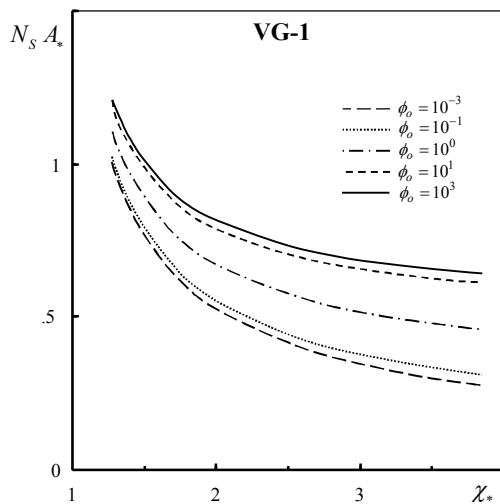Fig. 7(d). The variation of $N_S A_*$

with χ_* (elliptical ducts)

The variation of $N_S A_*$ with χ_* and ϕ_o as a parameter is shown in Figs. 7(b)-7(d). As seen, substantial improvement of the performance characteristics of the unit can be achieved by use of rectangular, Fig. 7(b), rectangular ducts with semicircular ends, Fig. 7(c), and elliptical ducts, Fig. 7(d). It is obviously that any unit designed with ducts of one of these non-circular shapes is superior to the unit with circular tubes.

In all these cases, the benefit increases with the gradually decrease of ϕ_o, and the best performance of

the unit can be achieved for $\phi_o = 10^{-3}$.

The unit with rectangular ducts with semicircular ends seems to be a little more attractive to the others.

Fig. 7(b). The variation of $N_S A_*$ with χ_* (rectangular ducts)

III.3.2. Case VG-2a

The objective functions of the case VG-2a are increased heat duty $Q_* > 1$, decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S / Q_* < 1$. The constraints imposed are: $W_* = 1$, $\Delta T_i^* = 1$, $P_* = 1$, $A_* = 1$, $A_f^* = 1$.

The consequences of the constraints are, $L_* < 1$, $N_* > 1$, $V_* < 1$.

The Eqs. (2)-(7) yield:

$$L_* = \left[\chi_*^2 \left(f\,Re \right)_* \right]^{-1/3} \qquad (36)$$

$$N_* = \chi_*^{1/6} \left(f\,Re \right)_*^{1/3} \qquad (37)$$

$$Q_* = \varepsilon_* \qquad (38)$$

$T_o^* = 0.981 + 0.019\varepsilon_*$, where ε_* is calculated by Eq. (10). $N_{S,P} = P_* = 1$, whereas:

$$N_{S,T} = \frac{Q_*^2}{Nu_* \chi_*^{2/3} \left(f\,Re \right)_*^{1/3}} \qquad (39)$$

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o} \left\{ \frac{Q_*^2}{Nu_* \chi_*^{2/3} \left(f\,Re \right)_*^{1/3}} + \phi_o \right\} \qquad (40)$$

The values of Q_*, calculated by Eq. (38) are presented in Fig. 8(a), as the variation of Q_* with χ_*.

As seen, the first objective $Q_* > 1$ can be achieved if $\chi_* > 1.8$.

If $\chi_* < 1.5$, the unit with elliptical ducts has better performance, whereas in the other case $\chi_* > 1.5$, the unit with rectangular ducts with semicircular ends brings about the greatest benefit.

The variation of N_S / Q_* with χ_* and ϕ_o as a parameter is shown in Figs. 8(b)-8(d). As seen, a great benefit benefit can be achieved if the unit is furnished with rectangular, Fig. 8(b), rectangular with semicircular ends, Fig. 8(c), or elliptical ducts. Fig. 8(d).

This case is very similar to the case VG-1. For all these non-circular ducts the improvement of the performance of the unit increases with the increase of χ_* and decrease of ϕ_o, and the best performance is reached for $\phi_o = 10^{-3}$. The unit furnished with rectangular ducts with semicircular ends is superior to the rest.
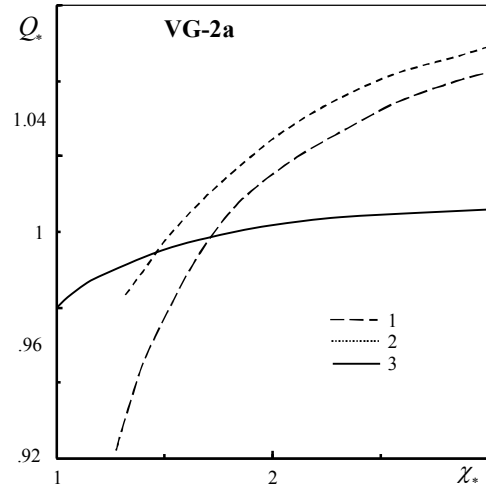

Fig. 8(a). The variation of Q_* with χ_* (1- rectangular ducts, 2- rectangular ducts with semicircular ends, 3- elliptical ducts)
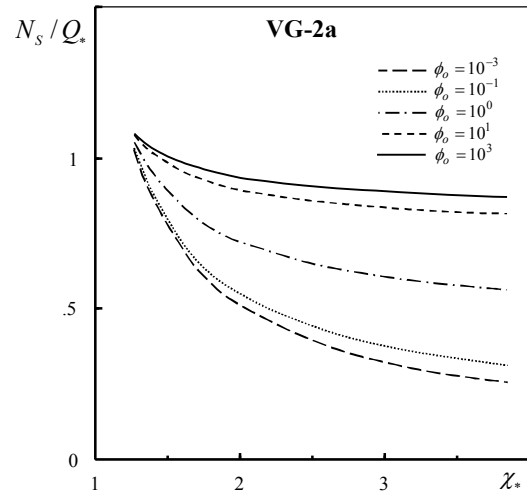

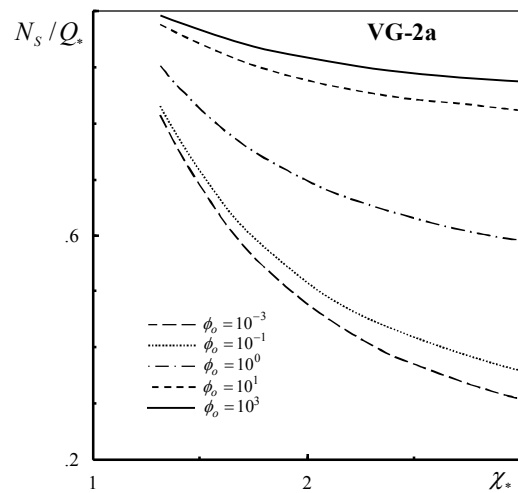Fig. 8(b). The variation of N_S / Q_* with χ_* (rectangular ducts)

Fig. 8(c). The variation of N_S / Q_* with χ_* (rectangular ducts with semicircular ends)
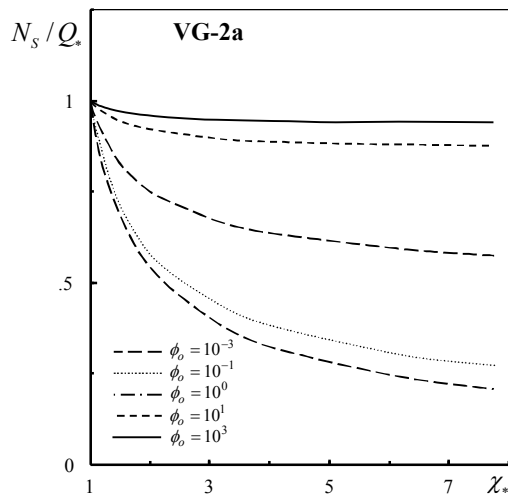
Fig. 8(d). The variation of $N_S P_*$ with χ_* (elliptical ducts)

IV. Discussion and Conclusions

In this study, a general criterion has been used as a combination of the objective functions based on first and second law of thermodynamics. This permits to overcome the contradictions in the evaluation of the benefit.

As a conclusion, the results of this study can be summarized as follows:

1. For the case FG-1a the use of ducts with non-circular shapes as studied can be more efficient only for very small values $\phi << 10^{-3}$ ($Be > 0.99$).

2. In the case FG-2a, for all non-circular ducts studied $Q_* < 1$ and Q_* decreases very fast with the increase of χ_*. However, when the two objectives are pursued simultaneously and general criterion N_S / Q_* is implemented some benefit can be achieved if $\phi_o \leq 0.1$, and it increases with the decrease of ϕ_o ($\phi_o << 1$).

3. In the case FG-3, the pumping power is always greater than unity, $P_* > 1$, irrespective of the values of ϕ_o and χ_*. It is obviously that the heat transfer performance of the heat exchanger cannot be improved by use of non-circular ducts and the advantage of implementation of circular tubes in the design of heat exchanger is unimpeachable.

4. In the case FN-1 the units with rectangular, rectangular ducts with semicircular ends or elliptical ducts possess almost one and the same performance characteristic $N_S A_* < 1$, and any one of these non-circular shapes is a better choice compare to the circular shape.

5. For the case FN-2 the use of non-circular ducts can bring about some benefit only if ϕ_o has small values, $\phi_o << 1$.

6. In the case VG-1, substantial improvement of the performance characteristics of the unit can be achieved by use of rectangular, rectangular ducts with semicircular ends or elliptical ducts. Any unit designed with ducts of one of these non-circular shapes is superior to the unit with circular tubes. The benefit increases with the decrease of ϕ_o, and the best performance of the unit can be achieved for $\phi_o = 10^{-3}$.

7. In the case VG-2a, a benefit can be achieved if the unit is furnished with rectangular, rectangular ducts with semicircular ends, or elliptical ducts. This case is very similar to the case VG-1. For all these non-circular ducts the improvement of the performance of the unit increases with the increase of χ_* and decrease of ϕ_o, and the best performance is reached for $\phi_o = 10^{-3}$.

As expected, the analysis of the thermal performance of heat exchangers with non-circular channels revealed that the selection of the optimal shape of the unit ducts strongly depends on the geometric and thermal-hydraulic constraints, and the objectives pursued. The use of a general criterion connecting two objectives simultaneously permits to avoid the contradictory results that can be obtained if criteria based on first or second law analysis are implemented alone.

The evaluation concerning the superiority of a particular shape of the non-circular duct depends also on the value of irreversibility distribution ratio, ϕ_o.

Accordingly, the selection of non-circular shape of the ducts as rectangular, rectangular with semicircular ends or elliptical can be beneficial if the irreversibility due to heat transfer across the wall of the reference channel dominates.

References

[1] R.L. Webb, Performance evaluation criteria for use of enhanced heat transfer surfaces in heat exchanger design, *Int. J. Heat Mass Transfer* 24 (1981) 715-726.

[2] R.L. Webb, A.E. Bergles, Performance evaluation criteria for selection of heat transfer surface geometries used in low Reynolds number heat exchangers, in Kakac, S., Shah, R.K., Bergles, A.E. (Eds) *Low Reynolds Number Flow Heat Exchangers,* Hemisphere Publ. Corp., Washington, D.C., 1983, pp. 735-742.

[3] A. Bejan, Entropy *Generation through Heat and Fluid Flow*, John Wiley & Sons, New York, 1982.

[4] A. Bejan, *Entropy Generation Minimization*, CRC Press, Boca Raton, 1996.

[5] V.D. Zimparov, Extended performance evaluation criteria for enhanced heat transfer surfaces: heat transfer through ducts with constant wall temperature, *Int. J. Heat Mass Transfer* 43 (2000) 3137-3155.

[6] V.D. Zimparov, Extended performance evaluation criteria for enhanced heat transfer surfaces: heat transfer through ducts with constant heat flux, *Int. J. Heat Mass Transfer* 44 (2001) 169-180.

[7] A. Z. Sahin, Thermodynamics of laminar viscous flow through a duct subjected to constant heat flux, *Energy* 21 (1996) 1179-1187.

[8] A. Z. Sahin, Entropy generation in turbulent liquid flow through a smooth duct subjected to constant wall temperature, *Int. J. Heat Mass Transfer* 43 (2000) 1469-1478.

[9] D. H. Richardson, D.P. Seculic, A. Campo, Low Reynolds number flow inside straight micro channels with irregular cross-sections, *Heat Mass Transfer* 36 (2000) 187-193.

[10] S.Y. Wu, Y.R. Li, Y. Chen, L. Xiao, Energy transfer characteristics of forced convective heat transfer through a duct with constant wall temperature, *Energy* 32 (2007) 2385-2395.

[11] H.F. Oztop, Effective parameters on second law analysis for semicircular ducts in laminar flow and constant wall heat flux, *Int. Comm. Heat Mass Transfer* 32 (2005) 266- 274.

[12] H.F. Oztop, I. Dagtekin, A.Z. Sahin, Second law analysis of fully developed laminar flow for rectangular ducts with semicircular ends, Int. Comm. Heat Mass Transfer 36 (2009) 725-730.

[13] A.Z. Sahin, Irreversibilities in various duct geometries with constant wall heat flux and laminar flow, *Energy* 23 (1997) 465-473.

[14] D.P. Sekulic, A. Campo, J.K. Morales, Irreversibility phenomena associated with heat transfer and fluid friction in laminar flows through singly connected ducts, *Int. J. Heat Mass Transfer* 40 (1997) 905-914.

[15] A.Z. Sahin, A second law comparison for optimum shape of duct subjected to constant wall temperature and laminar flow, *Heat Mass Transfer* 33 (1998) 425-430.

[16] E.B. Ratts, A.G. Raut, Entropy generation minimization of fully developed internal flow with constant heat flux, ASME *J. Heat Transfer* 126 (2004) 656-659.

[17] T.A. Jankowski, Minimizing entropy generation in internal flows by adjusting the shape of the cross section, *Int. J. Heat Mass Transfer* 52 (2009) 3439-3445.

[18] S. Jarungthammachole, Entropy generation analysis for fully developed laminar convection in hexagonal duct subjected to constant heat flux, *Energy* 35 (2010) 5374 - 5379.

[19] V.D. Zimparov, V.M. Petkov, Second law analysis of fully developed internal flows with different shape of the cross- section, (2011) *International Review of Chemical. Engineering* (*IRECHE*), 3 (2), pp. 256-264.

[20] S. Chakraborty, S. Ray, Performance optimization of laminar fully developed flow through square ducts with rounded corners, *Int. J. Therm. Sciences* 50 (2011) 2522-2535.

[21] S. Ray, D. Misra, Laminar fully developed flow through square and equilateral triangular ducts with rounded corners subjected to H1 and H2 boundary conditions, *Int. J. Therm. Sciences* 49 (2010) 1763-1775.

[22] V.M. Petkov, V.D. Zimparov, A.E. Bergles, Performance evaluation of ducts with non- circular shapes and laminar fully developed flow, (2012) *International Review of Chemical Engineering* (*IRECHE*), 4 (4), pp. 379-391.

[23] V.M. Petkov, V.D. Zimparov, A.E. Bergles, Performance Evaluation of Laminar Fully Developed Flow through Ducts With Non-Circular Shapes Subjected to H1 Boundary Condition. Part 1. (2013) *International Review of Chemical Engineering* (*IRECHE*), 5 (1), pp. 1-15.

[24] R.K. Shah, A.L. London, *Laminar Flow Forced Convection in Ducts*, 1978, Academic Press.

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