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Performance Evaluation of Laminar Fully Developed Flow through Ducts with Non-Circular Shapes Subjected to H1 Boundary Condition. Part 1

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Abstract – *Extended performance evaluation criteria (ExPEC) have been used to assess the performance characteristics of single-phase fully developed laminar flow through heat exchangers with axial corrugated, square with rounded corners, and triangular with rounded corners ducts. The heat exchanger with circular tubes has been used as a reference heat transfer unit. The H1 boundary condition has been selected as thermal boundary condition. The performance characteristics of the heat exchangers with non-circular tubes have been evaluated and compared to those of the reference unit for different objectives and constraints imposed. As a common constraint, the cross sectional area of the non-circular duct has been specified. As expected, the analysis of the thermal performance of heat exchangers with non-circular ducts revealed that the selection of the optimal shape of the duct strongly depends on the geometric and thermal-hydraulic constraints imposed on the unit, and the objectives pursued.*
The use of a general criterion connecting two objectives simultaneously permits to avoid the contradictory results that can be obtained if criteria based on first or second law analysis are implemented alone. The evaluation concerning the superiority of a particular shape of the non-circular duct depends also on the value of the irreversibility distribution ratio, ϕ_o. *For the cases FG (fixed geometry criteria) the use of ducts with non-circular shapes is inefficient, whereas in the cases FN (fixed number of tubes) and VG (variable geometry) some benefit can be achieved according to the value of* ϕ_o.

***Keywords**: Performance Evaluation Criteria, Single-Phase Laminar Flow, Non-Circular Ducts, Entropy Generation*

Nomenclature

A	Heat transfer surface area	[m^2]
c_p	Specific heat capacity	[$J\,kg^{-1}K^{-1}$]
D	Reference circular tube diameter	[m]
D_h	Hydraulic diameter	[m]
h	Heat transfer coefficient	[$W\,m^{-2}\,K^{1}$]
k	Thermal conductivity	[$W\,m^{-1}\,K^{-1}$]
L	Tube length	[m]
$\dot{m}$	Mass flow rate in tube	[$kg\,s^{-1}$]
N_t	Number of tubes	
P	Pumping power	[W]
p	Wetted perimeter	[m]
Δp	Pressure drop	[Pa]
$\dot{Q}$	Heat transfer rate	[W]
$\dot{S}_{gen}$	Entropy generation rate	[$W\,K^{-1}$]
T	Temperature	[K]
ΔT	Wall-to-fluid temperature difference	[K]
V	Volume of tubes	[m^3]
W	Mass flow rate in heat exchanger	[$kg\,s^{-1}$]
x	Axial distance along the tube	[m]

Greek symbols

ϑ	Temperature difference	$T_w - T$
μ	Dynamic viscosity	[Pa s]
ρ	Fluid density	[$kg\,m^{-3}$]

Dimensionless groups

A_*	Dimensionless heat transfer surface	$A_w / A_{w,c}$
D_*	Dimensionless tube diameter	D_h / D_c
L_*	Dimensionless tube length	L / L_c
f	Fanning friction factor	
f_*	Fanning friction factor ratio	f / f_c
Nu	Nusselt number	
Nu_*	Nusselt number ratio	Nu / Nu_c
N_S	Augmentation entropy generation number	

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N_*	Ratio of number of tubes	$N_t / N_{t,c}$
NTU	Heat transfer units	$4St\, L / D$
Pr	Prandtl number	
P_*	Dimensionless pumping power	P / P_c
Q_*	Dimensionless heat transfer rate	$\dot{Q} / \dot{Q}_c$
Re	Reynolds number	
Re_*	Reynolds number ratio	Re / Re_c
St	Stanton number	
ΔT_i^*	Dimensionless inlet temperature difference	$\Delta T_i / \Delta T_{i,c}$
V_*	Volume ratio	V / V_c
W_*	Dimensionless mass flow rate	W / W_c
χ	Shape factor	p / D_h
χ_*	Ratio of shape factors	χ / χ_c
ε_*	Ratio of heat exchanger effectiveness	$\varepsilon / \varepsilon_c$
τ	Dimensionless temperature difference	$\Delta T / T$
ϕ_o	Irreversibility distribution ratio	

Subscripts

c	Circular Tube
f	Fluid
i	Value at $x = 0$
m	Mean
o	Value at $x = L$
w	Wall

I. Introduction

In compact heat exchangers, due to smaller system dimensions, the hydraulic diameter is low, which, in most practical cases, yields essentially laminar flow. It is therefore important to investigate the performance characteristics of different ducted flows, particular in the laminar regime. Because of size and volume constraints in applications for aerospace, nuclear, biomedical engineering and electronics, chemical and process industries, it may be required to use non-circular flow-passage geometries, particularly in compact heat exchangers.

In most of the heat exchangers in service, especially in shell and tube type, circular duct is used. The primary objective of the heat exchanger designer is to use duct geometries that yield: (i) a high value of heat transfer area to volume ratio, (ii) a high value of heat transfer coefficient and (iii) a corresponding value of friction factor. Offering maximum compactness, i.e., highest surface area to volume ratio, however, is not enough for selection of duct geometry. A designer should also take into consideration the overall thermal hydraulic behavior of the flow through the ducts.

The performance of conventional heat exchangers, for single-phase flows in particular, can be substantially improved by many augmentation techniques, resulting in the design of high-performance thermal design systems. On the basis of the first-law analysis Webb and Bergles [1]-[2] have proposed performance evaluation criteria (PEC) that define the performance benefits of an exchanger having augmented surfaces, relative to standard exchanger with smooth surfaces subject to various objectives and design constraints.

On the other hand, it is well established that the minimization of the entropy generation in any process leads to the conservation of useful energy. A thermodynamic basis to evaluate the merit of augmentation techniques by second-law analysis has been proposed by Bejan [3]-[4] developing the entropy generation minimization (EGM) method. This method has been used as a general criterion for estimating and minimizing the irreversibilities and optimum-design method for heat exchangers.

The method has been extended by Zimparov [5]-[6] including the effect of fluid temperature variation along the length of a tubular heat exchanger, and new information has been added assessing two objectives simultaneously. The EGM method combined with the first law analysis provides the most powerful tool for the analysis of the thermal performance of any augmentation technique.

In many instances, the designer is faced with an existing equipment, where the space occupied by the cooling passage is at a premium and the heat and mass flow rates are limited by the size of an existing or retrofit pump or fan. In these situations, where a coolant passage must be designed so that the volume of the passage is restricted to some value and the heat rate and mass flow rate of the coolant are dictated by the available equipment, the designer may ask the question: Is there an optimal cross-sectional area and optimal length for the coolant passage that minimizes entropy generation and allows for the best performance?

A number of studies have been focused on the calculation and minimization of entropy generation in the fundamental fully-developed convective flow configuration in a duct. In most of these studies the entropy generation has been calculated and minimized in ducts with various cross-sectional shapes for laminar and turbulent configurations, with constant heat transfer rate per unit length, with constant heat flux, or with constant wall temperature, also in flows with temperature dependent physical properties [7]-[19].

Circular ducts are generally used in tubular heat exchangers and most of the augmentation techniques have been applied to such tubes.

As to the ducts with non-circular cross section the improvement of the performance of one geometry with respect to another depends on the duct geometry, inlet-to-wall temperature ratio and the operating Reynolds number for a given fluid and a certain duct length. Most of the analyses related to non-circular cross sections (namely, square, equilateral triangle, rectangular, sinusoidal, etc.) and with single-phase laminar flow have been conducted on the basis of the second law analysis.

In a recent paper, Chakraborty and Rey [20] have evaluated the performance of a square duct with rounded corners, for single-phase laminar flow using the PEC criteria identified by Webb and Bergles [1-2] taken as objective functions for each case, and second law analysis in an attempt to find out an optimal operating point, i.e., a particular radius of curvature for the corners, which is advantageous from both first and second law analyses. For any duct with non-circular shape, the size is determined by either the hydraulic diameter D_h, or the cross sectional area A_f, since these parameters are related through the shape factor $\chi = 4A_f / D_h^2$. In this regard, two different common constraints can be imposed – specified cross-sectional area $A_f^* = 1$, or specified hydraulic diameter of the ducts, $D_* = 1$.

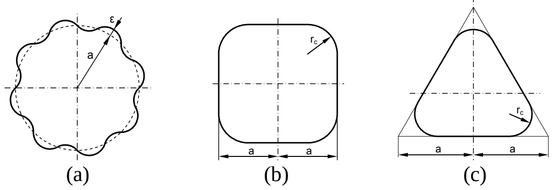
The results of performance evaluation criteria for the first case, $A_f^* = 1$, and constant wall temperature as a boundary condition, have been published recently by Petkov et al. [21].

The rationale of the study is to evaluate the thermal performance of tubular heat exchangers with non-circular cross section ducts as axial corrugated, square with rounded corners, triangular with rounded corners, and single-phase laminar flow in the ducts, Figs. 1. The boundary condition of the wall is H1 boundary condition. Similar to the study [21], the common constraint $A_f^* = 1$ is imposed. In this case, the ratio of diameters of the ducts is a consequence, $D_* = \chi_*^{-1/2}$.

The heat exchanger with circular tubes has been used as a reference heat transfer unit. Using the first and second laws simultaneously, the performance characteristics of units with non-circular tubes have been evaluated for different objectives and constraints imposed and compared to those of the reference unit.

II. Equations Based on the Entropy Production Theorem

Consider the energy and entropy balance of the general internal flow configuration, where fluid flows through a duct with a cross sectional area A_f, a perimeter p, and hydraulic diameter $D_h = 4A_f / p$.



(a) (b) (c)

Figs. 1. The geometrical parameters of non-circular ducts:
(a) axial corrugated duct, (b) square duct with rounded corners,
(c) triangular duct with rounded corners

The shape of the cross section is arbitrary but constant over the entire length of the duct. The flow is single-phase, fully developed laminar, incompressible and Newtonian.

The second law of thermodynamics for the elemental control volume can be written as:

$$d\dot{S}_{gen} = \dot{m}ds - \frac{q'dx}{T + \Delta T} \tag{1}$$

Since the cross-sectional area for the flow does not change in the axial direction, the mass flow rate ($\dot{m}_t$) through the duct remains constant. For H1 boundary condition (i.e., constant heat flow per unit axial length and uniform temperature along the periphery of the duct at a given axial location), q' is consider to be known.

Following the model developed by Zimparov [5] for tubular full-size heat exchangers, the rate of entropy generation can be expressed as:

$$\dot{S}_{gen} = \frac{\dot{Q}^2 D_h}{N_t^2 k_f pLNuT_iT_o} + \frac{8W^2 \mu L_t \left(f\,Re \right)}{N_t^2 \rho^2 pT_i D_h^3} \tag{2a}$$

or:

$$\dot{S}_{gen} = \frac{\dot{Q}^2}{N_t^2 k_f Nu\chi L_t T_i T_o} + \frac{8W^2 \left(f\,Re \right) \mu L_t}{N_t^2 \rho^2 \chi D_h^4 T_i} \tag{2b}$$

where:

$$\dot{Q} = N_t \dot{Q}_t,\ \dot{Q}_t = \dot{m}_t c_p \left(T_o - T_i \right),\ W = \dot{m}_t N_t$$

$$A = pLN_t,\ T_o = T_i + \frac{hp\Delta T}{\dot{m}_t c_p} L$$

(the heated and wetted perimeters are assumed to be the same.) The first and the second term on the right-hand side of Eq. (2) represent the entropy generation due to heat transfer across the finite temperature difference and due to fluid friction, respectively.

Following Bejan [3]-[4], the augmentation entropy generation number N_S can be presented as:

$$N_S = \frac{\dot{S}_{gen}}{\dot{S}_{gen,circle}} = \frac{N_{S,T} + \phi_o N_{S,P}}{1 + \phi_o} = \frac{1}{1 + \phi_o}\left(N_{S,T} + \phi_o N_{S,P} \right) \tag{3}$$

where [5]:

$$N_{S,T} = \frac{Q_*^2}{Nu_* N_*^2 \chi_* L_* T_o^*} \tag{3a}$$

$$T_o^* = \left[\frac{T_{i,c}}{T_{o,c}} + \frac{Q_*}{W_*}\left(1 - \frac{T_{i,c}}{T_{o,c}} \right) \right] \tag{3b}$$

$$N_{S,P} = \frac{W_*^2 L_* \left(f\, Re \right)_*}{N_*^2 \chi_* D_*^4} \equiv P_* \qquad (3c)$$

$$\phi_o = \frac{8 W_c^2 \mu L_c^2 \left(f\, Re \right)_c}{\rho^2 D_c^4} \times \frac{k_f Nu_c T_{o,c}}{\dot{Q}_c^2} \qquad (3d)$$

The numerical value of the irreversibility distribution ratio $\phi_o = \left(\dot{S}_{gen,\Delta P} / \dot{S}_{gen,\Delta T} \right)_c$, describes the thermodynamic mode in which the circular tube passage is meant to operate. This mode can be expressed also by the value of Bejan number, $Be = \left(\dot{S}_{gen,\Delta T} / \dot{S}_{gen} \right)_c$ or $Be = 1 / \left(1 + \phi_o \right)$. The PEC as suggested by Webb and Bergles [1]-[2] characterize nearly all the PEC and some of them will be considered below. The equations are developed for ducts with different cross sectional shape and the equations (in dimensionless form) for inside single-phase laminar flow are:

$$A_* = p_* L_* N_* = \chi_* D_* N_* L_* \qquad (4)$$

$$P_* = \frac{W_*^2 L_* \left(f\, Re \right)_*}{N_*^2 \chi_* D_*^4} \qquad (5)$$

$$Q_* = W_* \varepsilon_* \Delta T_i^* \qquad (6)$$

$$W_* = Re_*\, p_* N_* = Re_*\, \chi_* D_* N_* \qquad (7)$$

where $\chi_* = p_* / D_*$.

III. Performance Evaluation Criteria

The performance evaluation criteria, as suggested by Webb and Bergles [1]-[2] and extended by Zimparov [5] have been considered in this study. These criteria are based on the use of first and second law analyses in the pursuit of two objectives simultaneously.

In this study the geometrical and regime parameters of the reference channel (smooth circular tube) are selected to fulfill the requirement of $4L_c / \left(D_c\, Re\, Pr \right) = 1$.

The performance characteristics of heat exchangers with non-circular tubes, such as axial corrugated, square with rounded corners, and triangular with rounded corners shapes are compared to those of the reference circular tubes heat exchanger. The values of the shape factor χ, friction factor f, and Nusselt number Nu of non-circular channels are taken from Chakraborty and Ray [20], Ray and Misra [21], and Shah and London [23]. While obtaining the augmentation entropy generation number, the irreversibility distribution ratio for the circular configuration, ϕ_o, varies in the range $10^{-3} \le \phi_o \le 10^3$ ($0.001 < Be < 0.999$).

III.1. Fixed Geometry Criteria (FG)

These criteria involve a replacement of circular tubes by tubes with non-circular shape of equal length and cross sectional area. The FG-1 cases seek increased heat duty for constant exchanger flow rate and heat transfer area. The FG-2 criteria have the same objective as FG-1, but requires that the non-circular tube design to operate at the same pumping power as the reference circular tube design.

The third criterion, FG-3, attempts to effect reduced pumping power for constant heat duty and surface area.

III.1.1. Case FG-1a

The objective functions of the case FG-1a are increased heat rate $Q_* > 1$, decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them as a general performance criterion $N_S / Q_* < 1$.

The constraints imposed are: $W_* = 1$, $\Delta T_i^* = 1$, $A_* = 1$, $L_* = 1$, $A_f^* = 1$.

The consequences of these constraints are $D_* = \chi_*^{-1/2}$, $Re_* = 1$, $P_* > 1$, $N_* < 1$, $V_* < 1$.

The constraint of equal heat transfer surface area, $A_* = 1$, requires $N_* = \chi_*^{-1/2}$.

In this case the total bundle cross-sectional area and the volume ratio are $A_{f,tot}^* = N_* A_f^* = \chi_*^{-1/2}$, and $V_* = L_* A_{f,tot}^* = \chi_*^{-1/2}$. The constraint of equal mass flow rate $W_* = 1$, requires $Re_* = 1$.

The Eqs. (5) and (6) yield:

$$P_* = \chi_*^2 \left(f\, Re \right)_* \qquad (8)$$

and:

$$Q_* = \varepsilon_* \qquad (9)$$

where:

$$\varepsilon_* = 1.229 \frac{Nu}{1 + Nu} \qquad (10)$$

The Eqs. (3a)-(3c) yield:

$$T_o^* = 0.981 + 0.019 \varepsilon_* \qquad (11)$$

$$N_{S,T} = \frac{Q_*^2}{Nu_* T_o^*} \qquad (12)$$

$$N_{S,P} \equiv P_* = \chi_*^2 \left(f\, Re \right)_* \qquad (13)$$

and the augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o}\left(\frac{Q_*^2}{Nu_* T_o^*} + \phi_o \chi_*^2 \left(f\ Re\right)_*\right) \qquad (14)$$

The values of Q_* are calculated by Eq. (9), and the variation of Q_* with χ_* are shown in Fig. 2(a). As seen for all axial corrugated tubes studied $Q_* < 1$, and the goal $Q_* > 1$ cannot be achieved.

The value of Q_* decreases with the increase of χ_* and number of corrugations n.

The value of Q_* for heat exchangers with square ducts with rounded corners and triangular ducts with rounded corners is also $Q_* < 1$ and decreases very rapidly with the increase of χ_*.

The implementation of first and second law analysis simultaneously, as a general criterion N_S / Q_*, Figs. 2(b)-2(e), shows that the use of ducts with non-circular shapes as axial corrugated, square ducts with rounded corners and triangular with rounded corners is inefficient since the values of N_S / Q_* are always greater than unity.

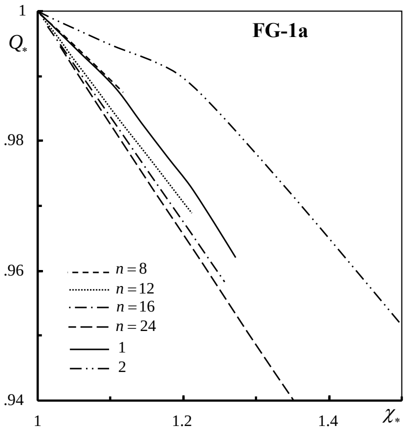


Fig. 2(a). The variation of Q_* with χ_* (axial corrugated tubes, 1- square ducts with rounded corners, 2- triangular ducts with rounded corners)

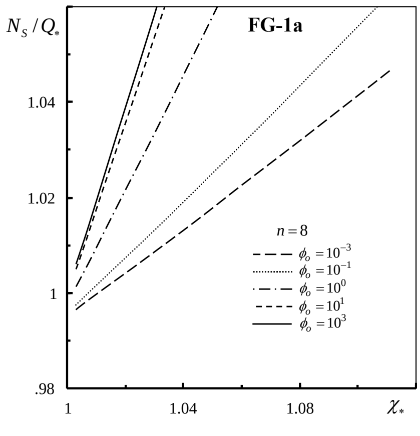


Fig. 2(b). The variation of N_S / Q_* with χ_* (axial corrugated tubes)

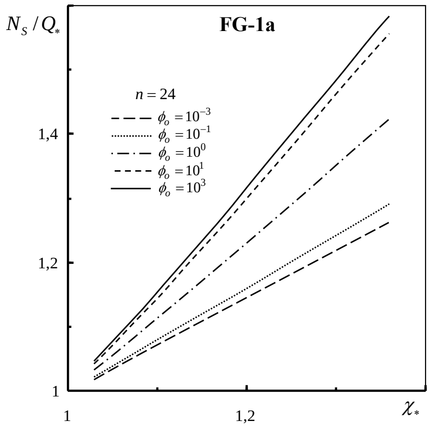


Fig. 2(c). The variation of N_S / Q_* with χ_* (axial corrugated tubes)

III.1.2. Case FG-2a

The objective functions of the case FG-2a are increased heat duty $Q_* > 1$, decreased entropy generation numbers $N_S < 1$, and simultaneous effect of the both of them $N_S / Q_* < 1$. The constraints imposed are: $\Delta T_i^* = 1$, $P_* = 1$, $A_* = 1$, $L_* = 1$, $A_f^* = 1$. The consequences of these constraints are $D_* = \chi_*^{-1/2}$, $W_* < 1$, $N_* < 1$. The constraint $A_* = 1$ requires $N_* = \chi_*^{-1/2}$, $A_{f,tot}^* = \chi_*^{-1/2}$ and $V_* = \chi_*^{-1/2}$. The constraint $P_* = 1$, requires:

$$W_* = Re_* = \chi_*^{-1}\left(f\ Re\right)_*^{-1/2} \qquad (15)$$

In this regard, Eqs. (2)-(7) yield:

$$Q_* = \frac{\varepsilon_*}{\chi_*\left(f\ Re\right)_*^{1/2}} \qquad (16)$$

$$N_{S,T} = \frac{Q_*^2}{Nu_* T_o^*} \qquad (17)$$

where ε_* and T_o^* are calculated by Eqs. (10) and (11).

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o}\left(\frac{Q_*^2}{Nu_* T_o^*} + \phi_o\right) \qquad (18)$$

The values of Q_* calculated by Eq. (16), are shown in Fig. 3(a). As seen, for all non-circular ducts studied, $Q_* < 1$ and it decreases very fast with the increase of χ_*.

However, when the two objectives are pursued simultaneously and general criterion N_S / Q_* is implemented in some cases benefit can be achieved, Figs. 3(b)-3(e).

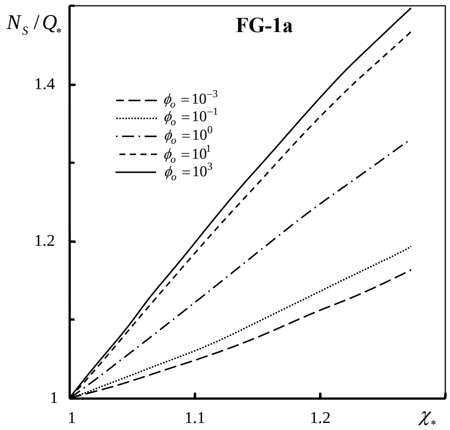


Fig. 2(d). The variation of N_S / Q_* with χ_*
(square ducts with rounded corners)

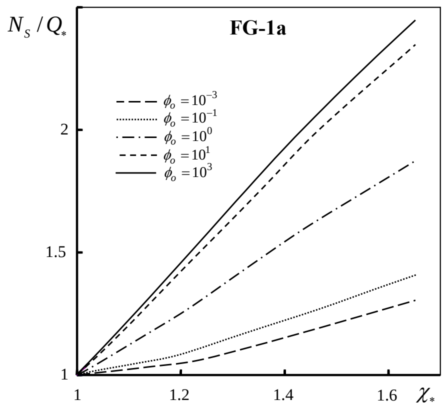


Fig. 2(e). The variation of N_S / Q_* with χ_*
(triangular ducts with rounded corners)

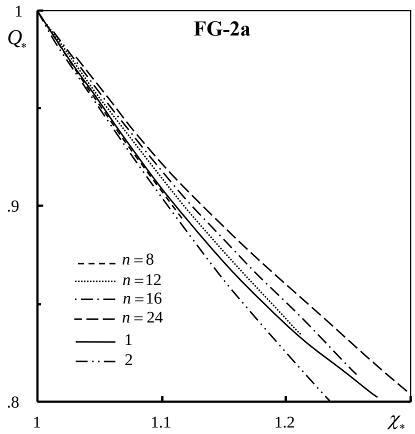


Fig. 3(a). The variation of Q_* with χ_* (axial corrugated tubes, 1-square ducts with rounded corners, 2- triangular ducts with rounded corners)

For axial corrugated ducts. Figs. 3(b)-3(c), the benefit can be obtained if $\phi_o < 1$, number of corrugations is small, $n = 8$, and the values of χ_* are near unity, $\chi_* \approx 1.02$. When n increases the benefit gradually decreases and disappears.

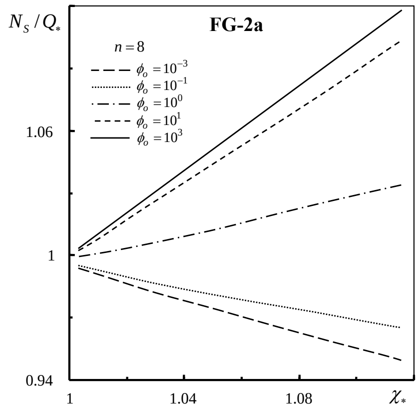


Fig. 3(b). The variation of N_S / Q_* with χ_*
(axial corrugated tubes)

For square ducts with rounded corners, Fig. 3(d), and triangular ducts with rounded corners, Fig. 3(e), the benefit can be achieved for small values of χ_*, $\phi_o < 1$, and with the decrease of ϕ_o ($\phi_o << 1$). It should be pointed out that $Q_* < 1$, and the benefit obtained due to the low values of N_S / Q_* should be restricted to small values of χ_* to sustain the value of Q_* as higher as possible.

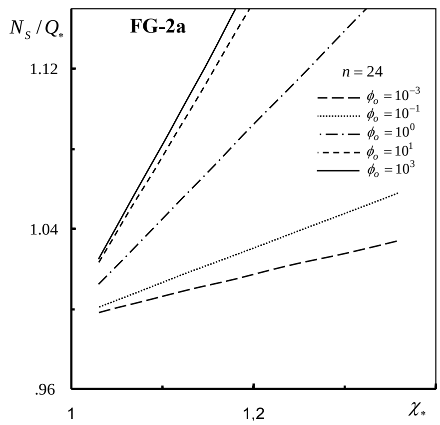


Fig. 3(c). The variation of N_S / Q_* with χ_* (axial corrugated tubes)

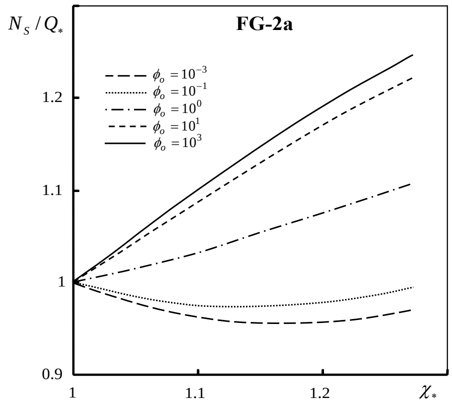


Fig. 3(d). The variation of N_S / Q_* with χ_*
(square ducts with rounded corners)

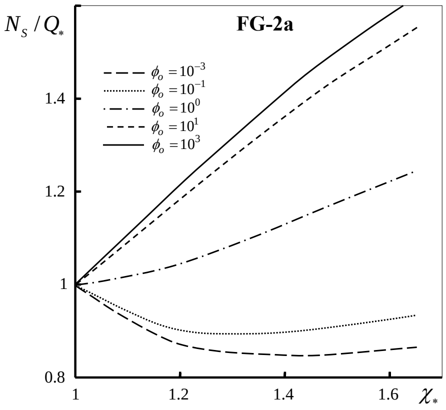


Fig. 3(e). The variation of N_S / Q_* with χ_* (triangular ducts with rounded corners)

III.1.2. Case FG-3

The objective functions of the case FG-3 are lower pumping power $P_* < 1$, decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S P_* < 1$. The constraints imposed are: $Q_* = 1$, $\Delta T_i^* = 1$, $A_* = 1$, $L_* = 1$, $A_f^* = 1$.

The consequences of these constraints are $D_* = \chi_*^{-1/2}$, $A_{f,tot}^* = V_* = \chi_*^{-1/2}$, and $N_* < 1$.

The Eqs. (2)-(7) yield $N_* = \chi_*^{-1}$, $W_* = Re_* = \varepsilon_*^{-1}$, where ε_* is calculated by Eq. (10):

$$P_* = \frac{\chi_*^2 \left(f\ Re \right)_*}{\varepsilon_*^2} \tag{19}$$

$T_o^* = 0.981 + 0.019\varepsilon_*$, $N_{S,P} = P_*$, and:

$$N_{S,T} = \frac{1}{Nu_* T_o^*} \tag{20}$$

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o} \left(\frac{1}{Nu_* T_o^*} + \phi_o \frac{\chi_*^2 \left(f\ Re \right)_*}{\varepsilon_*^2} \right) \tag{21}$$

The calculated values of P_* by Eq. (19) are presented in Fig. 4(a), whereas the results of using the general criterion $N_S P_*$ can be seen in the Figs. 4(b)-4(e).

It is obviously that in this case the heat transfer performance of the heat exchanger cannot be improved by use of non-circular ducts irrespective the values of ϕ_o and χ_*.

The advantage of implementation of circular tubes in the design of heat exchanger is unimpeachable.

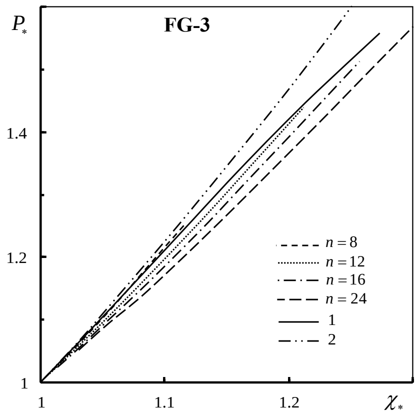


Fig. 4(a). The variation of P_* with χ_* (axial corrugated tubes, 1- square ducts with rounded corners, 2- triangular ducts with rounded corners)

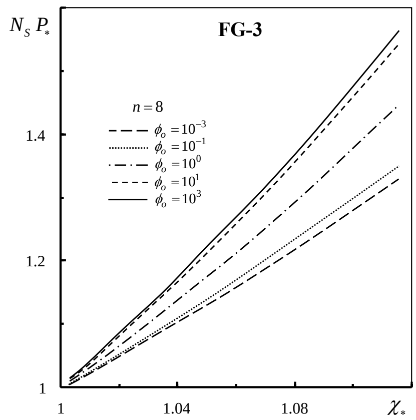


Fig. 4(b). The variation of $N_S P_*$ with χ_* (axial corrugated tubes)

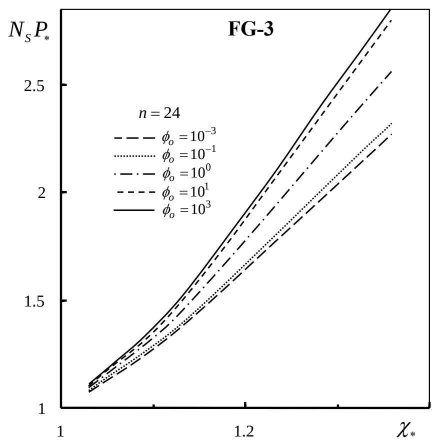


Fig. 4(c). The variation of $N_S P_*$ with χ_* (corrugated tubes)

III.2. Fixed Number of Tubes Criteria (FN)

These criteria maintain constant number of tubes. The objective of FN-1 is reduced surface area, via reduced tubing length, for constant pumping power. Reduced flow rate will probably be required to satisfy the constant pumping power criterion. The objective of FN-2 is reduced pumping power.

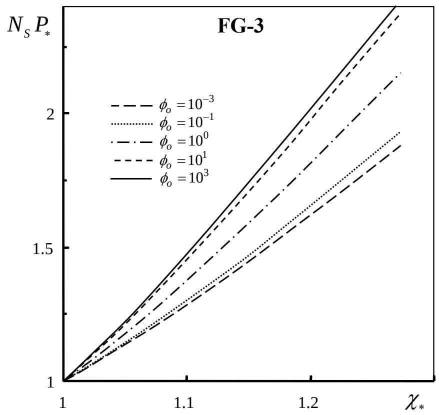


Fig. 4(d). The variation of $N_S P_*$ with χ_* (square ductswith rounded corners)

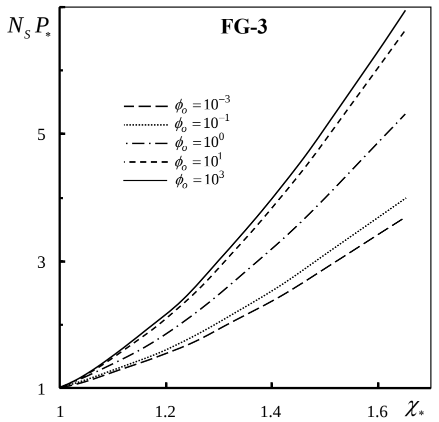


Fig. 4(e). The variation of $N_S P_*$ with χ_* (triangular ducts with rounded corners)

III.2.1. Case FN-1

The objective functions of the case FN-1 are reduced heat transfer area $A_* < 1$ ($L_* < 1$), decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S A_* < 1$.

The constraints imposed are: $N_* = 1$, $Q_* = 1$, $P_* = 1$, $\Delta T_i^* = 1$, and $A_f^* = 1$.

The consequences of the constraints are $D_* = \chi_*^{-1/2}$, $L_* < 1$, $A_{f,tot}^* < 1$, $V_* < 1$.

The Eqs. (2)-(7) yield:

$$W_* = \chi_* Re_* = \varepsilon_*^{-1} \tag{22}$$

$$L_* = \frac{\varepsilon_*^2}{\chi_* \left(f\, Re \right)_*} \tag{23}$$

$$A_* = N_* \chi_* L_* D_* = \frac{\varepsilon_*^2}{\chi_*^{1/2} \left(f\, Re \right)_*} \tag{24}$$

where ε_* is calculated by Eq. (10), $N_{S,P} = P_* = 1$, $T_o^* = 0.981 + 0.019\varepsilon_*$:

$$N_{S,T} = \frac{\left(f\, Re \right)_*}{Nu_* T_o^* \varepsilon_*^2} \tag{25}$$

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o} \left(\frac{\left(f\, Re \right)_*}{Nu_* T_o^* \varepsilon_*^2} + \phi_o \right) \tag{26}$$

The calculated values of A_* by Eq. (24) are shown in Fig. 5(a). The variations of A_* with χ_* clearly show that the use of non-circular ducts can substantially diminish the required heat transfer area.

The implementation of the general criterion $N_S A_*$, however, cools down the first optimistic expectations.

The use of axial corrugated ducts, for instance, Figs. 5(b)-5(c), can bring about some benefit if the number of corrugations is small ($n = 8$) and $\phi_o > 0.1$, Fig. 5(b).

The benefit increases with the increase of χ_* and ϕ_o. When the number of corrugations increases, Fig. 5(c), the benefit gradually decreases and can only be obtained at higher values of ϕ_o, $\phi_o > 1$.

If the heat exchanger is designed with square ducts with rounded corners, Fig. 5(d), the benefit can be achieved only if $\phi_o \geq 1$ and it increases with the increase of χ_*. This benefit, however, cannot exceed 8%. The use of triangular ducts with rounded corners, Fig. 5(e), can also bring about some benefit, but only for values $\phi_o \geq 1$ and this benefit cannot exceed 20%. The curve of $N_S A_*$ clearly shows the presence of a minimum and it moves on the right with the increase of χ_*.

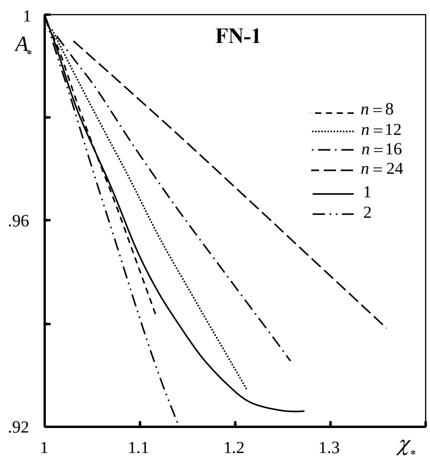


Fig. 5(a). The variation of A_* with χ_* (axial corrugated tubes, 1- square ducts with rounded corners, 2- triangular ducts with rounded corners)

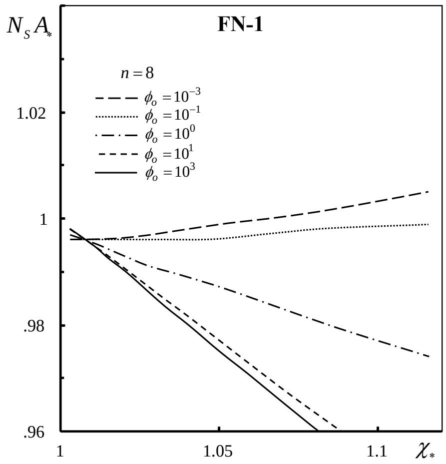


Fig. 5(b). The variation of $N_S A_*$ with χ_* (axial corrugated tubes)

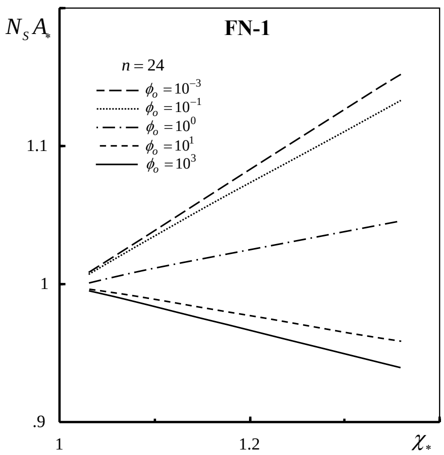


Fig. 5(c). The variation of $N_S A_*$ with χ_* (axial corrugated tubes)

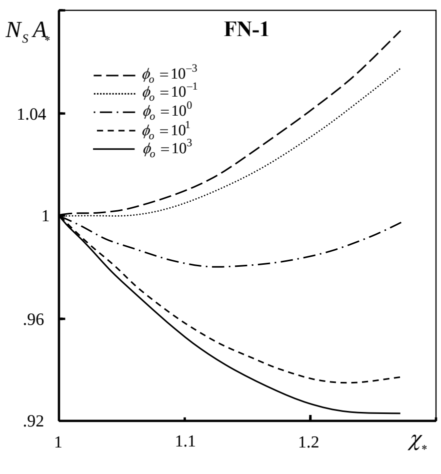


Fig. 5(d). The variation of $N_S A_*$ with χ_* (square ducts with rounded corners)

III.2.2. Case FN-2

The objective functions of the case FN-2 are lower pumping power $P_* < 1$, decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S P_* < 1$. The constraints imposed are: $Q_* = 1$, $A_* = 1$, $\Delta T_i^* = 1$, $N_* = 1$, and $A_f^* = 1$.

The consequences of the constraints are $D_* = \chi_*^{-1/2}$, $L_* < 1$, $A_{f,tot}^* < 1$, $V_* < 1$.

Eqs. (2)-(7) yield $L_* = V_* = \chi_*^{-1/2}$, $W_* = \varepsilon^{-1}$, $Re_* = \left(\chi_*^{1/2}\varepsilon_*\right)^{-1}$:

$$P_* = \frac{\chi_*^{1/2}\left(f\,Re\right)_*}{\varepsilon_*^2} \tag{27}$$

$T_o^* = 0.981 + 0.019\varepsilon_*$, where ε_* is calculated by Eq. (10). $N_{S,P} = P_*$, whereas:

$$N_{S,T} = \frac{1}{Nu_* \chi_*^{1/2} T_o^*} \tag{28}$$

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o}\left[\frac{1}{Nu_* \chi_*^{1/2} T_o^*} + \phi_o \frac{\chi_*^{1/2}\left(f\,Re\right)_*}{\varepsilon_*^2}\right] \tag{29}$$

The calculated values of P_* by Eq. (27) are shown in Figs. 6(b)-6(e). As seen, the variation of P_* with χ_* revealed that the first objective $P_* < 1$ cannot be achieved by the use of the non-circular ducts as studied.

On the other hand, however, the non-circular ducts can substantially decrease the generated entropy of the unit and the implementation of the general criterion $N_S P_* < 1$ shows that benefit can be obtained, particularly for $\phi_o \le 1$.

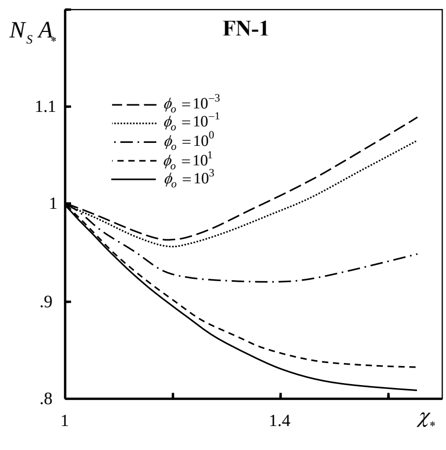


Fig. 5(e). The variation of $N_S A_*$ with χ_* (triangular ducts with rounded corners)

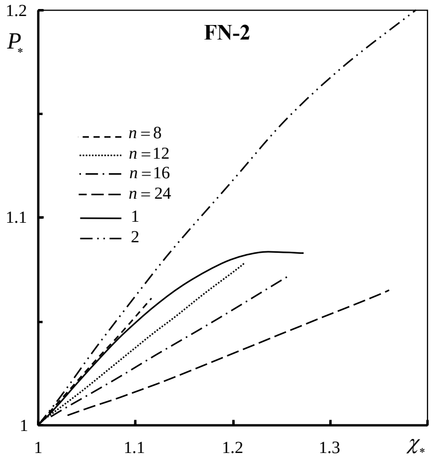


Fig. 6(a). The variation of P_* with χ_* (axial corrugated tubes, 1- square ducts with rounded corners, 2- triangular ducts with rounded corners)

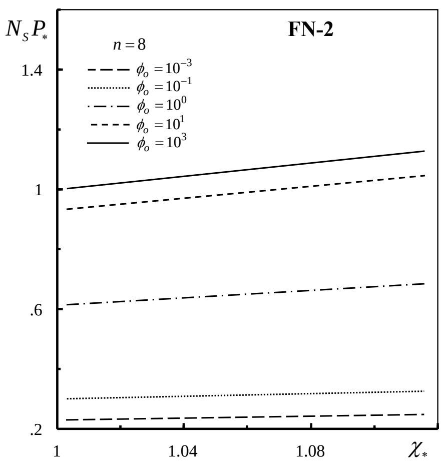


Fig. 6(b). The variation of $N_S P_*$ with χ_* (axial corrugated tubes)

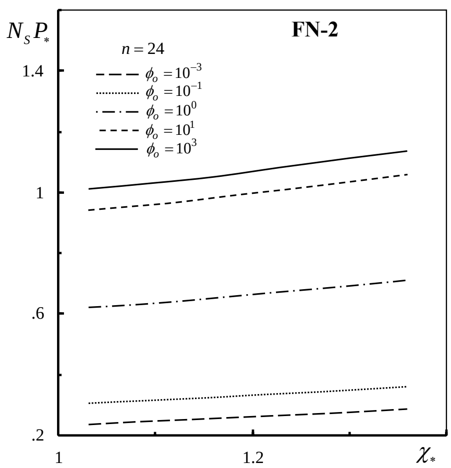


Fig. 6(c). The variation of $N_S P_*$ with χ_* (corrugated tubes)

For instance, for axial corrugated ducts, Figs. 6(b)-6(c), the values of $N_S P_*$ nearly do not depend on the number of corrugations n and benefit can be achieved if $\phi_o \leq 1$. It increases with the decrease of ϕ_o and the values of $N_S P_*$ approach 0.23-0.25 for $\phi_o = 10^{-3}$ and $n = 8$. If square ducts with rounded corners are used in the unit, Fig. 6(d), for $\phi_o \leq 1$ $N_S P_* < 1$ and it decreases with the decrease of ϕ_o.

Another interesting behavior of the curves is that in the vicinity of $\chi_* = 1$ they steep and reach minimum at $\chi_* \simeq 1.003$, and after that the value of $N_S P_*$ very slow increases.

The greatest benefit can be obtained for $\phi_o << 1$ ($\phi_o = 10^{-3}$).

The variation of $N_S P_*$ with χ_* for triangular ducts with rounded corners, Fig. 6(e), has the same tendency as the unit with square ducts with rounded corners and the results are almost the same.

Consequently, for the case FN-2 the use of non-circular ducts can bring about some benefit only if $\phi_o << 1$.

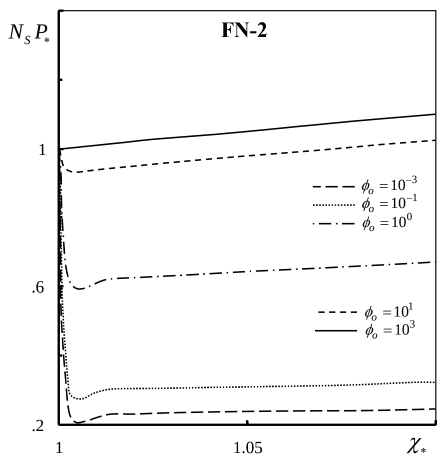


Fig. 6(d). The variation of $N_S P_*$ with χ_* (square ducts with rounded corners)

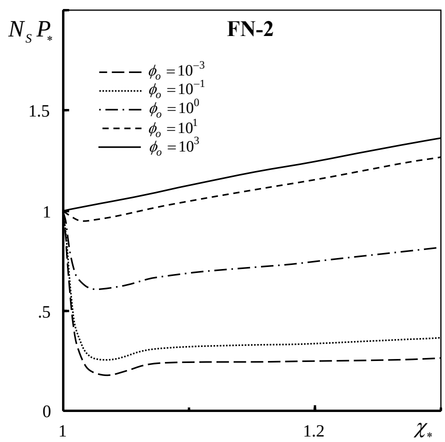


Fig. 6(e). The variation of $N_S P_*$ with χ_* (triangular ducts with rounded corners)

III.3. Variable Geometry Criteria (VG)

The criteria VG are applicable when the heat exchanger is "sized" for a required thermal duty with specified flow rate.

III.3.1. Case VG-1

The objective functions of the case VG-1 are lower heat transfer area $A_* < 1$, decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S A_* < 1$.

The constraints imposed are: $Q_* = 1$, $W_* = 1$, $P_* = 1$, $\Delta T_i^* = 1$, $A_f^* = 1$. The consequences of the constraints are $D_* = \chi_*^{-1/2}$, $L_* < 1$, $A_{f,tot}^* > 1$, $V_* < 1$. The Eqs. (2)-(7) yield: $\varepsilon_* = 1$, $T_o^* = 1$, $N_{S,P} = P_* = 1$:

$$A_* = Nu_*^{-1} \tag{30}$$

(since $Q_* = 1$, and $\Delta T_m^* = 1$):

$$N_* = \left[\frac{\chi^{1/2}\left(f\,Re\right)_*}{Nu_*}\right]^{1/3} \tag{31}$$

$$L_* = \left[\chi_*^2\left(f\,Re\right)_* Nu_*^2\right]^{-1/3} \tag{32}$$

$$Re_* = \chi_*^{1/6}\left(f\,Re\right)_*^{1/3} Nu_*^{2/3} \tag{33}$$

$$N_{S,T} = \left[\frac{Nu_*}{\chi_*^2\left(f\,Re\right)_*}\right]^{1/3} \tag{34}$$

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o}\left\{\frac{Nu_*^{1/3}}{\chi_*^{2/3}\left(f\,Re\right)_*^{1/3}} + \phi_o\right\} \tag{35}$$

The calculated values of A_* by Eq. (30) are shown in Fig. 7(a) as the variation of A_* with χ_*. As seen, the the first objective $A_* < 1$ cannot be achieved by using of axial corrugated tubes, square ducts with rounded corners and triangular ducts with rounded corners.

The variation of $N_S A_*$ with χ_* and ϕ_o as a parameter is shown in Figs. 7(b)-7(e). As seen, the benefit that can be obtained by the use of axial corrugated ducts, Figs. 7(b)-7(c), square ducts with rounded corners, Fig. 7(d), and triangular ducts with rounded corners, Fig. 7(e), is quite limited and can be obtained only for values $\phi_o << 1$.

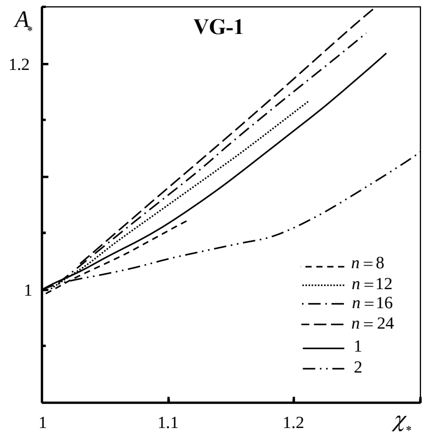


Fig. 7(a). The variation of A_* with χ_* (axial corrugated tubes, 1-square ducts with rounded corners, 2- triangular ducts with rounded corners)

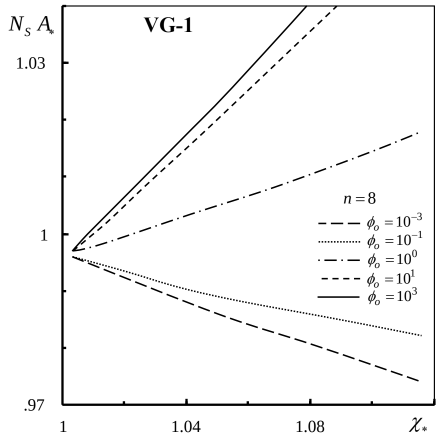


Fig. 7(b). The variation of $N_S A_*$ with χ_* (axial corrugated tubes)

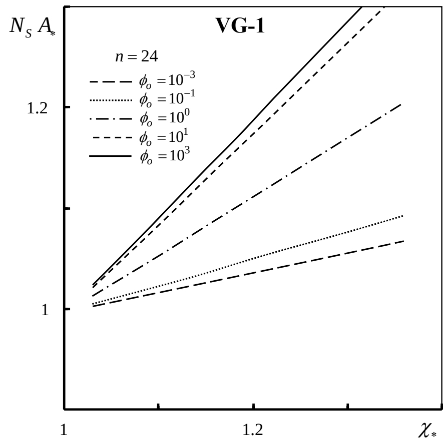


Fig. 7(c). The variation of $N_S A_*$ with χ_* (axial corrugated tubes)

III.3.2. Case VG-2a

The objective functions of the case VG-2a are increased heat duty $Q_* > 1$, decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S / Q_* < 1$. The constraints imposed are: $W_* = 1$, $\Delta T_i^* = 1$, $P_* = 1$, $A_* = 1$, $A_f^* = 1$.

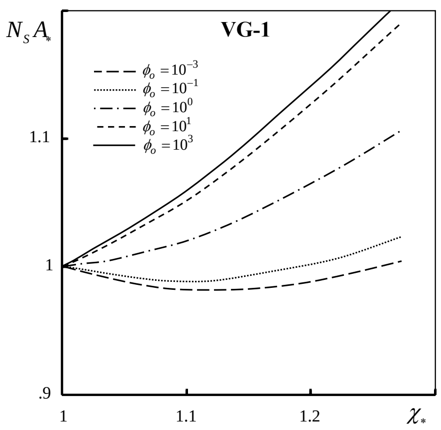


Fig. 7(d). The variation of $N_S A_*$ with χ_* (square ducts with rounded corners)

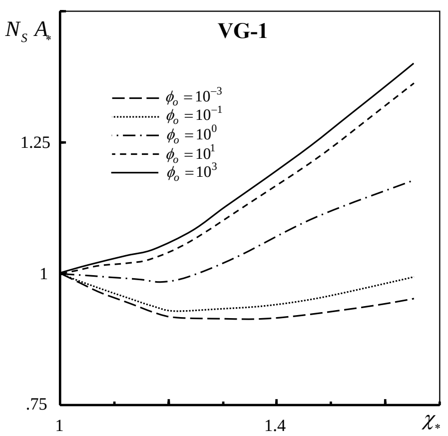


Fig. 7(e). The variation of $N_S A_*$ with χ_* (triangular ducts with rounded corners)

The consequences of the constraints are, $L_* < 1$, $N_* > 1$, $V_* < 1$.

The Eqs. (2)-(7) yield:

$$L_* = \left[\chi_*^2 \left(f\,Re \right)_* \right]^{-1/3} \tag{36}$$

$$N_* = \chi_*^{1/6} \left(f\,Re \right)_*^{1/3} \tag{37}$$

$$Q_* = \varepsilon_* \tag{38}$$

$T_o^* = 0.981 + 0.019\varepsilon_*$, where ε_* is calculated by Eq. (10). $N_{S,P} = P_* = 1$, whereas:

$$N_{S,T} = \frac{Q_*^2}{Nu_* \chi_*^{2/3} \left(f\,Re \right)_*^{1/3}} \tag{39}$$

The augmentation entropy generation number N_S becomes:

$$N_S = \frac{1}{1+\phi_o} \left\{ \frac{Q_*^2}{Nu_* \chi_*^{2/3} \left(f\,Re \right)_*^{1/3}} + \phi_o \right\} \tag{40}$$

The values of Q_*, calculated by Eq. (38) are presented in Fig. 8(a), as the variation of Q_* with χ_*. As seen, similarly to the previous case, the first objective $Q_* > 1$ cannot be achieved if the unit is designed with axial corrugated tubes, square ducts with rounded corners or triangular ducts with rounded corners.

The variation of N_S / Q_* with χ_* and ϕ_o as a parameter is shown in Figs. 8(b)-8(e). As seen, the benefit that can be obtained by the use of axial corrugated ducts, Figs. 8(b)-8(c), square ducts with rounded corners, Fig. 8(d), and triangular ducts with rounded corners, Fig. 8(e), is very small and can be obtained only for values of $\phi_o << 1$.

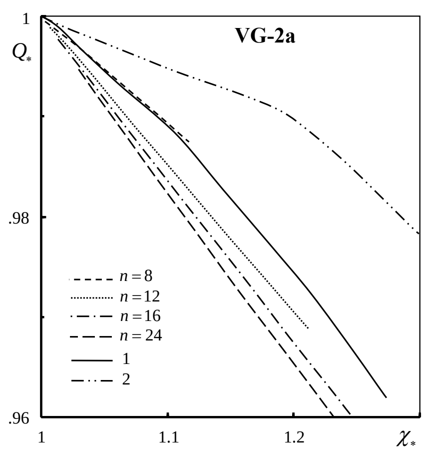
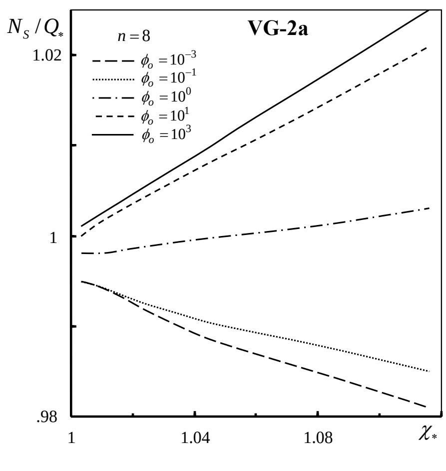


Fig. 8(a). The variation of Q_* with χ_* (axial corrugated tubes, 1- square ducts with rounded corners, 2- triangular ducts with rounded corners)



Figs. 8(b). The variation of N_S / Q_* with χ_* (axial corrugated tubes)

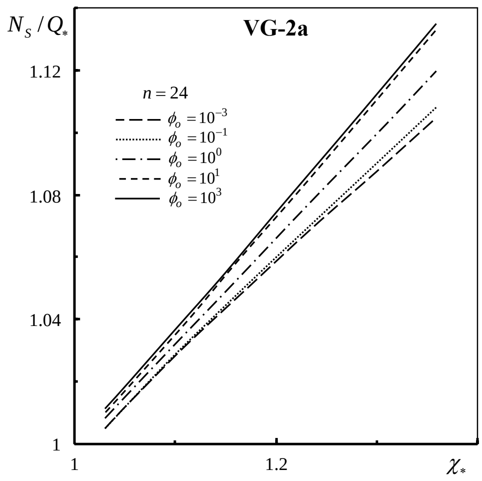


Fig. 8(c). The variation of N_S / Q_* with χ_* (axial corrugated tubes)

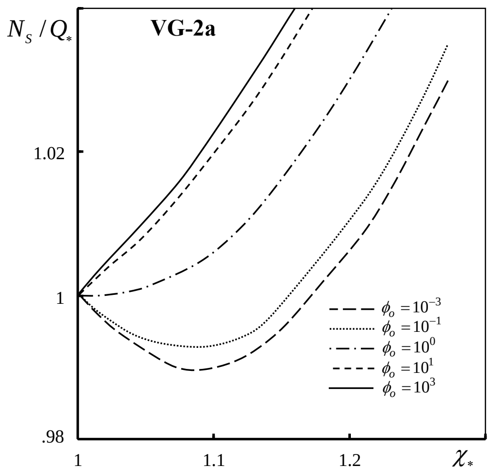


Fig. 8(d). The variation of N_S / Q_* with χ_* (square ducts with rounded corners)

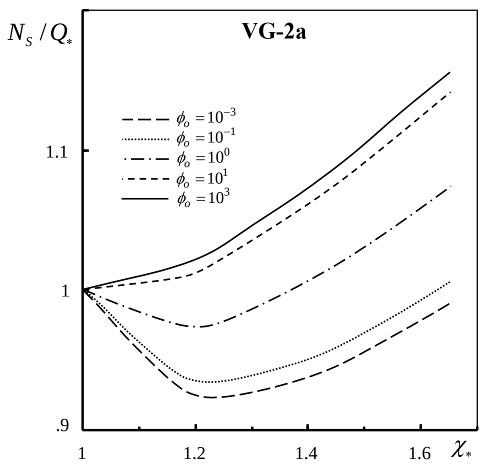


Fig. 8(e). The variation of N_S / Q_* with χ_* (triangular ducts with rounded corners)

IV. Discussion and Conclusion

The results obtained in this study for square ducts with rounded corners can be compared with those reported by Chakraborty and Ray [20] (Duct-IV , the same cross-sectional area of the ducts, $A_f^* = 1.0$). The general difference between the two studies is that Chakraborty and Ray [20] assessed and compared the performance characteristics of two single ducts: square duct with rounded corners and circular duct, whereas in our study, the constraints have been imposed on the heat transfer unit (bundle of ducts).

On the other hand, Chakraborty and Ray [20] assessed the benefit of the implementation of square duct with rounded corners instead of circular duct by two separate objective functions – one based on the first law analysis and entropy generation number as another objective function, based on the second law analysis. In some cases, this may lead to contradictory results and it may appear that the second law based criterion is probably inappropriate for evaluation of performances.

In this study, a general criterion has been used as a combination of the objective functions based on the first and second law of thermodynamics. This permits to overcome the contradictions in the evaluation of the benefit. In this respect, the results for the benefits in this study differ from those of Chakraborty and Ray [20].

For instance, a review of the results reported by Chakraborty and Ray [20] revealed that for case FG-1a the objective function Q_*, Fig. 4(a) [20], is always greater than unity which implies that the rounded corner square ducts perform better than the circular one. On the other hand, the augmentation entropy generation number N_S is greater than unity, Fig. 4(b) [20], and there is no conclusion weather the rounded corner square duct performs better than the circular one or not. In our study, for case FG-1a $Q_* < 1$, Fig. 2(b), and N_S / Q_* is always greater than unity irrespective the value of ϕ_o.

This clearly indicates that the performance of the unit with square ducts with rounded corners is poor as compared to the unit with circular ducts. If the reader goes carefully through the two studies on the subject he will find out many other differences in the evaluation of the benefits. In the end, we can summarize the results of this study as follows:

1. For the case FG-1a the use of ducts with non-circular shapes as studied is inefficient since the general criterion $N_S / Q_* > 1$.
2. In the case FG-2a, for all non-circular ducts studied $Q_* < 1$ and Q_* decreases very fast with the increase of χ_*. However, when the two objectives are pursued simultaneously and general criterion N_S / Q_* is implemented some benefit can be achieved if $\phi_o \le 0.1$, and it increases with the decrease of ϕ_o ($\phi_o << 1$).

3. In the case FG-3, the pumping power is always greater than unity, $P_* > 1$, irrespective of the values of ϕ_o and χ_*. It is obviously that the heat transfer performance of the heat exchanger cannot be improved by use of non-circular ducts and the advantage of implementation of circular tubes in the design of heat exchanger is unimpeachable.
4. In the case FN-1, the use of non-circular ducts as studied in the unit can bring about some benefit only if $\phi_o \geq 1$.
5. For the case FN-2 the use of non-circular ducts can bring about some benefit only if ϕ_o has small values, $\phi_o << 1$.
6. In the case VG-1, the benefit that can be obtained is quite limited and only for values $\phi_o << 1$.
7. In the case VG-2a, a benefit that can be achieved is very small only for values of $\phi_o << 1$.

As expected, the analysis of the thermal performance of heat exchangers with non-circular channels revealed that the selection of the optimal shape of the unit ducts strongly depends on the geometric and thermal-hydraulic constraints, and the objectives pursued. The use of a general criterion connecting two objectives simultaneously permits to avoid the contradictory results that can be obtained if criteria based on first or second law analysis are implemented alone. The evaluation concerning the superiority of a particular shape of the non-circular duct depends also on the value of irreversibility distribution ratio, ϕ_o.

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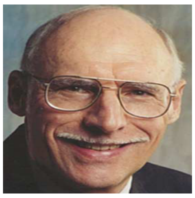

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