

## Performance Evaluation of Ducts with Non-Circular Shapes and Laminar Fully Developed Flow

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**Abstract** – Extended performance evaluation criteria (ExPEC) have been used to assess the performance characteristics of single-phase fully developed laminar flow through heat exchangers with rectangular, isosceles triangular, elliptical and hexagonal ducts. The heat exchanger with circular tubes has been used as a reference heat transfer unit. The constant wall temperature has been selected as thermal boundary condition. The performance characteristics of the heat exchangers with non-circular tubes have been evaluated and compared to those of the reference unit for different objectives and constraints imposed. As a common constraint, the cross sectional area of the non-circular duct has been specified. The results from this study have clearly showed that, in some cases, the rectangular and elliptical duct configuration can compete the circular tube unit. The choice of the duct shape depends on the constraints imposed and the objectives pursued. The results obtained from the present study have confirmed again the importance of the use of ExPEC (two objectives to be pursued simultaneously) to assess the benefits and select the optimal heat exchanger design. **Copyright © 2012 Praise Worthy Prize S.r.l. - All rights reserved.**

**Keywords:** Performance Evaluation Criteria, Single-Phase Laminar Flow, Non-Circular Ducts, Entropy Generation

### Nomenclature

$A$	Heat transfer surface area [m <sup>2</sup> ]	$\vartheta$	Temperature difference, $T_w - T$
$c_p$	Specific heat capacity [J kg <sup>-1</sup> K <sup>-1</sup> ]	$\mu$	Dynamic viscosity [Pa s]
$D$	Reference circular tube diameter [m]	$\rho$	Fluid density [kg m <sup>-3</sup> ]
$D_h$	Hydraulic diameter [m]	<i>Dimensionless groups</i>	
$h$	Specific enthalpy [J kg <sup>-1</sup> ]	$A_*$	Dimensionless heat transfer surface, $A_w / A_{w,c}$
$k$	Thermal conductivity [W m <sup>-1</sup> K <sup>-1</sup> ]	$D_*$	Dimensionless tube diameter, $D_h / D_c$
$L$	Tube length [m]	$L_*$	Dimensionless tube length, $L / L_c$
$\dot{m}$	Mass flow rate in tube [kg s <sup>-1</sup> ]	$f$	Fanning friction factor
$N_t$	Number of tubes	$f_*$	Fanning friction factor ratio, $f / f_c$
$P$	Pumping power [W]	$Nu$	Nusselt number
$p$	Wetted perimeter [m]	$Nu_*$	Nusselt number ratio, $Nu / Nu_c$
$\Delta p$	Pressure drop [Pa]	$N_S$	Augmentation entropy generation number
$\dot{Q}$	Heat transfer rate [W]	$N_*$	Ratio of number of tubes, $N_t / N_{t,c}$
$\dot{S}_{gen}$	Entropy generation rate [W K <sup>-1</sup> ]	$NTU$	Heat transfer units, $4StL/D$
$T$	Temperature [K]	$Pr$	Prandtl number
$\Delta T$	Wall-to-fluid temperature difference [K]	$P_*$	Dimensionless pumping power, $P / P_c$
$V$	Volume of tubes [m <sup>3</sup> ]	$Q_*$	Dimensionless heat transfer rate, $\dot{Q} / \dot{Q}_c$
$W$	Mass flow rate in heat exchanger [kg s <sup>-1</sup> ]	$Re$	Reynolds number
$x$	Axial distance along the tube [m]	$Re_*$	Reynolds number ratio, $Re / Re_c$
<i>Greek symbols</i>		$St$	Stanton number
$\alpha$	Heat transfer coefficient [W m <sup>-2</sup> K <sup>-1</sup> ]	$\Delta T_i^*$	Dimensionless inlet temperature difference, $\Delta T_i / \Delta T_{i,c}$

$V_*$	Volume ratio, $V/V_c$
$W_*$	Dimensionless mass flow rate, $W/W_c$
$\mathcal{X}$	Shape factor, $p/D_h$
$\mathcal{X}_*$	Ratio of shape factors, $\mathcal{X}/\mathcal{X}_c$
$\varepsilon_*$	Ratio of heat exchanger effectiveness, $\varepsilon/\varepsilon_c$
$\tau$	Dimensionless temperature difference, $\Delta T/T$
$\phi_o$	Irreversibility distribution ratio

#### Subscripts

$c$	Circular tube
$f$	Fluid
$i$	Value at $x = 0$
$m$	Mean
$o$	Value at $x = L$
$w$	Wall

## I. Introduction

Efficient utilization of exergy is one of the primary objectives in designing any thermodynamic system. Consequently, there is increased need for utilization of a variety of duct geometries for heat transfer applications with forced convection and internal flow. Because of size and volume constraints in applications to aerospace, nuclear, biomedical engineering and electronics, it may be required to use non-circular flow-passage geometries, particularly in compact heat exchangers. The performance of conventional heat exchangers, for single-phase flows in particular, can be substantially improved by many augmentation techniques resulting in the design of high-performance thermal design systems. Heat transfer enhancement devices are commonly employed to improve the performance of an existing heat exchanger or to reduce the size and cost of a proposed heat exchanger. On the basis of the first-law analysis Webb and Bergles [1]-[2] have proposed performance evaluation criteria (PEC) which define the performance benefits of an exchanger having enhanced surfaces, relative to standard exchanger with smooth surfaces subject to various objectives and design constraints.

On the other hand, it is well established that the minimization of the entropy generation in any process leads to the conservation of useful energy. A thermodynamic basis to evaluate the merit of augmentation techniques by second-law analysis has been proposed by Bejan [3]-[4] developing the entropy generation minimization (EGM) method. This method has been used as a general criterion for estimating and minimizing the irreversibilities and optimum-design method for heat exchangers. The coupling between fluid flow and heat transfer irreversibilities suggested that the geometry and operating conditions can be optimized to minimize the overall entropy generation. The method has been extended by Zimparov [5]-[6] including the effect of fluid temperature variation along the length of a

tubular heat exchanger, and new information has been added assessing two objectives simultaneously. The EGM method combined with the first law analysis provides the most powerful tool for the analysis of the thermal performance of any augmentation technique.

In many instances, the designer is faced with an existing equipment, where the space occupied by the cooling passage is at a premium and the heat and mass flow rates are limited by the size of an existing or retrofit pump or fan. In these situations, where a coolant passage must be designed so that the volume of the passage is restricted to some value and the heat rate and mass flow rate of the coolant are dictated by the available equipment, the designer may ask the question: Is there an optimal cross-sectional area and optimal length for the coolant passage that minimizes entropy generation and allows for the best performance?

A number of studies have been focused on the calculation and minimization of entropy generation in the fundamental fully-developed convective flow configuration in a duct. In most of these studies the entropy generation has been calculated and minimized in ducts with various cross-sectional shapes for laminar and turbulent configurations, with constant heat transfer rate per unit length, with constant heat flux, or with constant wall temperature, also in flows with temperature dependent physical properties [7]-[18].

Circular ducts are generally used in tubular heat exchangers and most of the augmentation techniques have been applied on such tubes. As to the ducts with non-circular cross section the improvement of the performance of one geometry with respect to another depends on the duct geometry, inlet-to-wall temperature ratio and the operating Reynolds number for a given fluid and a certain duct length. The most of analyses related to non-circular cross sections (namely, square, equilaterally triangle, rectangular, sinusoidal etc.) and single-phase laminar flow have been conducted on the basis of the second law analysis. In a recent paper, Chakraborty and Rey [19] have evaluated the performance of a square duct with rounded corners, for single-phase laminar flow using the PEC criteria identified by Webb and Bergles [1]-[2] taken as objective functions for each case, and second law analysis in an attempt to find out an optimal operating point, i.e., a particular radius of curvature for the corners, which is advantageous from both first and second law analyses. This study is a new evidence of the statement that the first and second law analyses should be used simultaneously to assess the thermal performance of the heat transfer units. The rationale of this study is to complement the work made by others on the thermal performance of the tubular heat exchangers with non-circular cross sections and a single-phase laminar flow in the channels. The heat exchanger with circular tubes has been used as a reference heat transfer unit. Using the first and second law simultaneously the performance characteristics of units with non-circular tubes have been

evaluated for different objectives and constraints imposed and compared to those of the reference unit.

The boundary condition of the wall is a constant wall temperature.

## II. Equations Based on the Entropy Production Theorem

Consider the energy and entropy balance of the general internal flow configuration, where fluid flows through a duct with a cross sectional area  $A_f$ , a perimeter  $p$ , and hydraulic diameter  $D_h = 4A_f / p$ . The shape of the cross section is arbitrary but constant over the entire length of the duct. The flow is single-phase, fully developed laminar, incompressible with Newtonian fluid.

Following the model developed by Zimparov [5], for tubular full-size heat exchanger, the rate of entropy generation can be expressed as:

$$\dot{S}_{gen} = \frac{\dot{Q}}{N_t} \frac{(T_w - T_o)}{T_i T_o} + \frac{32W^3 f}{N_t^3 \rho^2 p^2 D_h^3} \frac{L_t}{T_w} \quad (1)$$

or:

$$\dot{S}_{gen} = \frac{\dot{Q}}{N_t} \frac{g_o}{T_i T_o} + \frac{8W^2 (f Re) \mu L_t}{N_t^2 \rho^2 p D_h^3} \frac{L_t}{T_w} \quad (1-a)$$

where  $\dot{Q} = N_t \dot{Q}_t$ ,  $W = \dot{m}_t N_t$ ,  $A = p L N_t$ , and:

$$T_w - T_o = (T_w - T_i) \exp(-NTU) = \Delta T_i \exp(-NTU) \quad (1-b)$$

The first and the second term on the right-hand side of Eq. (1) represent the entropy generation due to heat transfer across the finite temperature difference and due to fluid friction, respectively.

Following Bejan [3]-[4], the augmentation entropy generation number can be presented as:

$$\begin{aligned} N_S &= \frac{\dot{S}_{gen}}{\dot{S}_{gen,circle}} = \frac{N_{S,T} + \phi_o N_{S,P}}{1 + \phi_o} \\ &= \frac{1}{1 + \phi_o} (N_{S,T} + \phi_o N_{S,P}) \end{aligned} \quad (2)$$

where [5]:

$$N_{S,T} = \frac{Q_*}{N_*} \frac{g_o}{g_{o,c}} \frac{T_{o,c}}{T_o} = \frac{Q_* g_o}{N_* T_o} \quad (2-a)$$

$$T_o^* = \left[ \frac{T_{i,c}}{T_{o,c}} + \frac{Q_*}{W_*} \left( 1 - \frac{T_{i,c}}{T_{o,c}} \right) \right] \quad (2-b)$$

$$g_o^* = \exp \left[ NTU_c \left( 1 - \frac{St}{St_c} \frac{L_*}{D_*} \right) \right] \quad (2-c)$$

$$N_{S,P} = \frac{W_*^2 L_* (f Re)_*}{N_*^2 p_* D_*^3} \frac{T_{w,c}}{T_w} \equiv P_* \quad (2-d)$$

$$\phi_o = \frac{8\dot{m}_c \mu L_c}{\pi \rho^2 D_c^4 c_p} \frac{T_{i,c} T_{o,c}}{g_{o,c} (T_o - T_i)_c T_w} (f Re)_c \quad (2-e)$$

The numerical value of the irreversibility distribution ratio  $\phi_o = (\dot{S}_{gen,\Delta P} / \dot{S}_{gen,\Delta T})_c$  describes the thermodynamic mode in which the circular tube passage is meant to operate.

The PEC as suggested by Webb and Bergles [1]-[2] characterizes nearly all the PEC and they will be considered below. The equations are developed for tubes with different cross sectional shape and the relative equations for inside single-phase laminar flow are:

$$A_* = p_* L_* N_* = \chi_* D_* N_* L_* \quad (3)$$

$$P_* = \frac{W_*^2 L_* (f Re)_*}{N_*^2 \chi_* D_*^4} \quad (4)$$

$$Q_* = W_* \varepsilon_* \Delta T_i^* \quad (5)$$

$$W_* = Re_* p_* N_* = Re_* \chi_* D_* N_* \quad (6)$$

where  $\chi_* = p_* / D_*$ .

## III. Performance Evaluation Criteria

The performance evaluation criteria, as suggested by Webb and Bergles [1]-[2] and extended by Zimparov [5] have been considered in this study. These criteria are based on the use of first and second law analyses in the pursuit of two objectives simultaneously. In this study the geometrical and regime parameters of the reference channel (smooth circular tube) are selected to fulfill the requirement of  $4L_c / (D_c Re Pr) = 1$ .

The performance characteristics of heat exchangers with non-circular ducts, such as rectangular, isosceles triangular, elliptical and hexagonal shapes are compared to those of the reference circular tube heat exchanger.

The values of the shape factor  $\chi$ , friction factor  $f$ , and Nusselt number  $Nu$  of non-circular channels are taken from Shah and London [20]. The irreversibility distribution ratio for the circular configuration,  $\phi_o$ , has varied in the range  $10^{-3} \leq \phi_o \leq 10^3$ . For any duct, the size is determined by either the hydraulic diameter or the cross sectional area, since these parameters are related

through the shape factor  $\chi = 4A_f / D_h^2$ . In this study, the cross sectional area of the ducts is specified,  $A_f^* = 1$  and the ratio of hydraulic diameters is a consequence,  $D_* = \chi_*^{-1/2}$ . The case of specified hydraulic diameter,  $D_* = 1$ , and the results of performance evaluation criteria will be consider in another study.

### III.1. Fixed Geometry Criteria (FG)

These criteria involve a replacement of circular tubes by tubes with non-circular shape of equal length and cross sectional area. The FG-1 cases seek increased heat duty for constant exchanger flow rate and heat transfer area.

The FG-2 criteria have the same objective as FG-1, but requires that the non-circular tube design to operate at the same pumping power as the reference circular tube design. The third criterion, FG-3, attempts to effect reduced pumping power for constant heat duty and surface area.

#### III.1.1. Case FG-1a

The objective functions of the case FG-1a are increased heat rate  $Q_* > 1$ , decreased entropy generation number  $N_S < 1$ , and simultaneous effect of the both of them as a general performance criterion  $N_S / Q_* < 1$ . The constraints imposed are:  $W_* = 1$ ,  $\Delta T_i^* = 1$ ,  $A_* = 1$ ,  $A_f^* = 1$ ,  $L_* = 1$ .

The consequences of these constraints are  $D_* = \chi_*^{-1/2}$ ,  $V_* < 1$ ,  $Re_* = 1$ ,  $P_* > 1$  and  $N_* < 1$ .

The constraint of equal heat transfer surface area,  $A_* = N_* \chi_* D_* L_* = 1$ , requires  $N_* \chi_*^{-1/2} = 1$  or  $N_* = \chi_*^{-1/2}$ .

The total bundle cross-sectional area is  $A_{f,tot}^* = N_* A_f^*$  or  $A_{f,tot}^* = \chi_*^{-1/2}$ , and also the volume ratio  $V_* = L_* A_{f,tot}^*$  or  $V_* = \chi_*^{-1/2}$ . The constraint of equal mass flow rate,  $W_* = Re_* D_* N_* \chi_* = 1$ , requires  $Re_* = 1$ .

The relative Eqs. (4) and (5) yield:

$$P_* = \frac{W_*^2 L_* (f Re)_*}{N_*^2 \chi_* D_*^4} = \chi_*^2 (f Re)_* \quad (7)$$

$$Q_* = W_* \varepsilon_* \Delta T_i^* = \varepsilon_* \quad (8)$$

where:

$$\varepsilon_* = 1.0265 \left[ 1 - \exp(-Nu \chi_*^{1/2}) \right] \quad (9)$$

The relative Eqs. (2-a)-(2-d) yield:

$$T_o^* = 0.981 + 0.019 Q_*$$

$$g_o^* = \exp \left[ 3.657 \left( 1 - Nu \chi_*^{1/2} \right) \right] \quad (10)$$

$$N_{S,T} = \frac{Q_* g_o^*}{N_* T_o^*} = \chi_*^{1/2} Q_* \frac{g_o^*}{T_o^*} \quad (11)$$

$$N_{S,P} = P_* = \chi_*^2 (f Re)_* \quad (12)$$

The general performance criterion becomes:

$$N_S / Q_* = \frac{\chi_*^{1/2}}{1 + \phi_o} \left( \frac{g_o^*}{T_o^*} + \phi_o \frac{\chi_*^{3/2} (f Re)_*}{Q_*} \right) \quad (13)$$

The calculated values of  $Q_*$  by Eq. (9), are shown in Fig. 1(a) and as seen they varied as follows: for heat exchanger with rectangular ducts  $Q_* = 0.991 - 1.026$ , for isosceles triangular ducts  $Q_* = 0.984 - 0.996$ , for elliptical ducts  $Q_* = 1.0 - 1.026$ , and for hexagonal duct  $Q_* = 0.996$ . Consequently, rectangular, elliptical and hexagonal ducts behave at the same way as circular ducts, or a little better, whereas for the isosceles triangular ducts  $Q_* < 1$  and decreases with the decrease of  $\chi_*$ . Figs. 1(b)-1(d) present the variation of  $N_S / Q_*$  with  $\chi_*$  and  $\phi_o$  as a parameter for rectangular, elliptical, hexagonal and isosceles triangular ducts.

Several important conclusions can be made from these figures:

(i) The performance of heat exchanger with rectangular channels (Fig. 1(b)) depends on the value of  $\phi_o$ , and the benefit can be achieved only for  $\phi_o < 1$  and  $\chi_* > 1.35$ . The smaller  $\phi_o$  the greater benefit. The curves have a minimum which goes to the right when  $\chi_*$  increases. The heat exchanger with rectangular channels and aspect ratio  $a/b = 16$  realizes the greatest efficiency.

(ii) Heat exchanger with elliptical ducts (Fig. 1(c)) performs similar as heat exchanger with rectangular ducts but with a smaller efficiency. Heat exchanger with hexagonal ducts is inefficient.

(iii) The design of heat exchanger with isosceles triangle ducts (Fig. 1(d)) is completely inefficient since  $N_S / Q_* > 1$  for all values of  $\chi_*$  and  $\phi_o$  and this inefficiency increases with the increase of  $\chi_*$  and  $\phi_o$ .

#### III.1.2. Case FG-2a

The objective functions of the case FG-2a are increased heat duty  $Q_* > 1$ , decreased entropy generation numbers  $N_S < 1$ , and simultaneous effect of the both of them  $N_S / Q_* < 1$ .

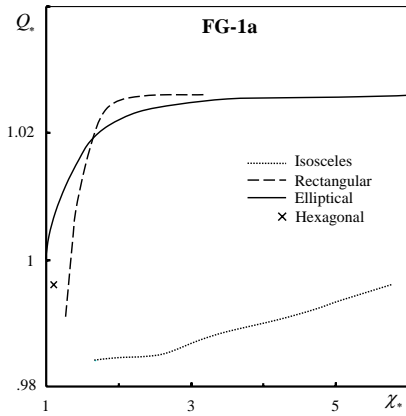


Fig. 1(a). The variation of  $Q_*$  with  $\chi_*$

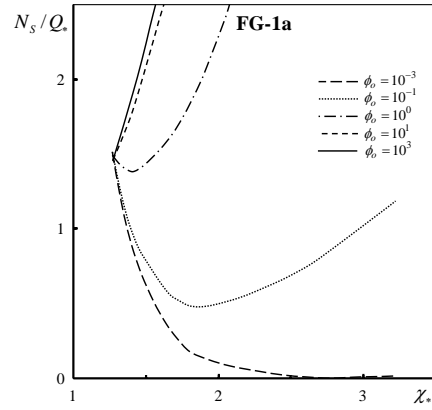


Fig. 1(b) The variation of  $N_s / Q_*$  with  $\chi_*$  (rectangular ducts)

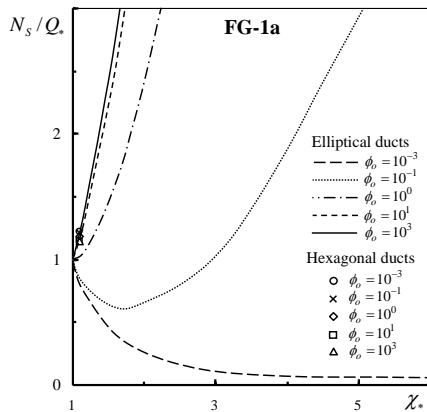


Fig. 1(c). The variation of  $N_s / Q_*$  with  $\chi_*$  (elliptical and hexagonal ducts)

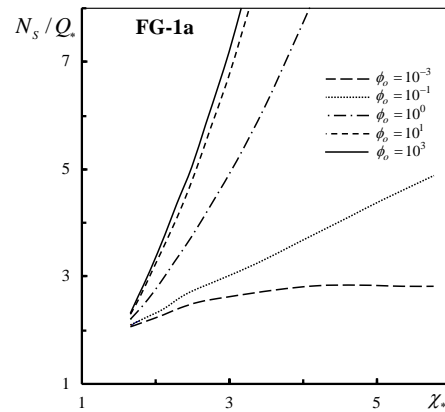


Fig. 1(d). The variation of  $N_s / Q_*$  with  $\chi_*$  (isosceles triangular ducts)

The constraints imposed are:  $\Delta T_i^* = 1$ ,  $P_* = 1$ ,  $A_* = 1$ ,  $A_f^* = 1$ ,  $L_* = 1$ . The consequences of these constraints are  $D_* < 1$ ,  $W_* < 1$ ,  $N_* < 1$ , and  $V_* < 1$ .

The constraint  $A_f^* = 1$  requires  $D_* = \chi_*^{-1/2}$ , whereas  $A_* = 1$  requires  $N_* = \chi_*^{-1/2}$  and  $V_* = \chi_*^{-1/2}$ ;  $W_* = Re_*$  is obtained from the constraint  $P_* = 1$ :

$$W_* = Re_* = \chi_*^{-1} (f Re)_*^{-1/2} \quad (14)$$

In this regard, Eqs. (2)-(6) yield:

$$Q_* = \varepsilon_* \chi_*^{-1} (f Re)_*^{-1/2} \quad (15)$$

where:

$$\varepsilon_* = 1.0265 \left\{ 1 - \exp \left[ - \left( Nu \chi_*^{3/2} (f Re)_*^{1/2} \right) \right] \right\} \quad (16)$$

$$N_{S,T} = \frac{\chi_*^{1/2} Q_* \mathcal{G}_o^*}{T_o^*}, N_{S,P} = P_* = 1 \quad T_o^* = 0.981 + 0.019 \varepsilon_*$$

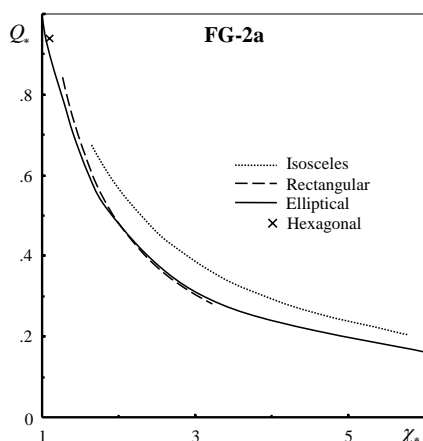
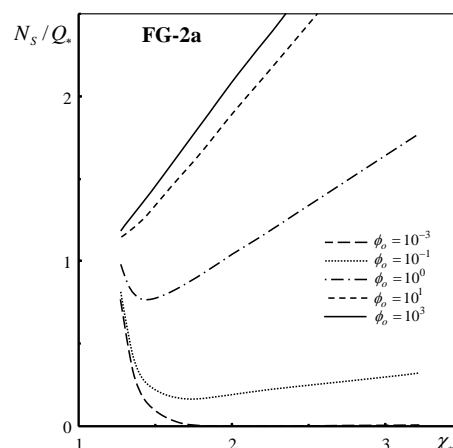
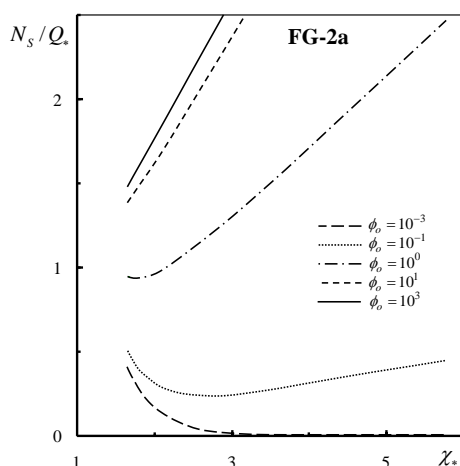
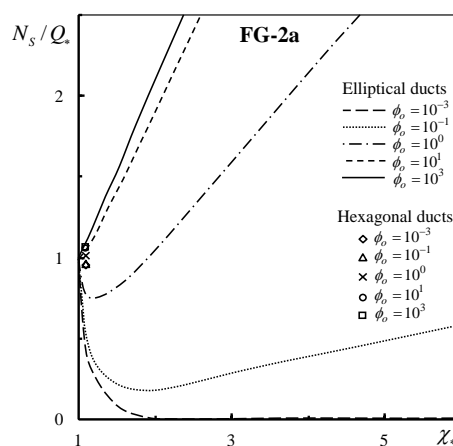
$$\mathcal{G}_o^* = \exp \left\{ 3.657 \left[ 1 - Nu_* \chi_*^{3/2} (f Re)_*^{1/2} \right] \right\} \quad (17)$$

The general performance criterion becomes:

$$N_s / Q_* = \frac{1}{1 + \phi_o} \left( \frac{\chi_*^{1/2} \mathcal{G}_o^*}{T_o^*} + \frac{\phi_o}{Q_*} \right) \quad (18)$$

The calculated values of  $Q_*$  by Eq. (15), are shown in Fig. 2(a) and as seen they varied as follows: for heat exchanger with rectangular ducts  $Q_* = 0.842 - 0.282$ , for isosceles triangular ducts  $Q_* = 0.676 - 0.203$ , for elliptical ducts  $Q_* = 1.0 - 0.148$ , and for hexagonal duct  $Q_* = 0.940$ . The constraint  $P_* = 1$  requires substantial decrease of mass flow rate and number of tubes, and the goal  $Q_* > 1.0$  cannot be achieved for all non-circular channels.

Figs. 2(b)-2(d) show the variation of general criterion  $N_s / Q_*$  with  $\chi_*$  and  $\phi_o$  as a parameter. As seen, the heat exchanger with rectangular channels (Fig. 2b) can be efficient for  $\phi_o < 1$  and the benefit increases with the decrease of  $\phi_o$ .


 Fig. 2(a). The variation of  $Q_*$  with  $\chi_*$ 

 Fig. 2(b). The variation of  $N_S / Q_*$  with  $\chi_*$  (rectangular ducts)

 Fig. 2(c). The variation of  $N_S / Q_*$  with  $\chi_*$  (isosceles triangular ducts)

 Fig. 2(d). The variation of  $N_S / Q_*$  with  $\chi_*$  (elliptical and hexagonal ducts)

The curves  $N_S / Q_* = f(\chi_*)$  show a minimum which goes to the right with the increase of  $\chi_*$  and decrease of  $\phi_o$ .

However, it should take into account that this benefit is due to the smaller heat flow transferred by rectangular ducts,  $Q_* < 1$ . For  $\phi_o > 1$  the use of rectangular ducts is totally inefficient. Heat exchangers with elliptical ducts (Fig. 2(c)) and isosceles triangular ducts (Fig. 2(d)) perform in the same way as heat exchanger with rectangular ducts but with a smaller efficiency. The efficiency of heat exchanger with hexagonal ducts is close to the unit with circular tubes but not better.

### III.1.3. Case FG-3

The objective functions of the case FG-3 are lower pumping power  $P_* < 1$ , decreased entropy generation number  $N_S < 1$ , and simultaneous effect of the both of them  $N_S P_* < 1$ . The constraints imposed are:  $Q_* = 1$ ,  $\Delta T_i^* = 1$ ,  $A_* = 1$ ,  $A_f^* = 1$ ,  $L_* = 1$ . The consequences of

these constraints are  $D_* < 1$ ,  $N_* < 1$  and  $V_* < 1$ . The constraint  $A_f^* = 1$  requires  $D_* = \chi_*^{-1/2}$ , whereas  $A_* = 1$  requires  $N_* = \chi_*^{-1/2}$  and  $V_* = \chi_*^{-1/2}$ .

The relative Eqs. (2)-(6) yield:  $W_* = Re_* = \varepsilon_*^{-1}$ , where:

$$\varepsilon_* = 1.0265 \left[ 1 - \exp(-Nu \chi_*^{1/2} \varepsilon_*) \right] \quad (19)$$

Eq. (19) is to be solved by an iterative process for any  $Nu$ . In this case:

$$P_* = \frac{\chi_*^2 (f Re)_*}{\varepsilon_*^2} \quad (20)$$

$$\mathcal{G}_o^* = \exp \left[ 3.657 \left( 1 - Nu_* \varepsilon_* \chi_*^{1/2} \right) \right] \quad (21)$$

$$T_o^* = 0.981 + 0.019 \varepsilon_*$$

$$N_{S,T} = \chi_*^{1/2} \frac{g_o^*}{T_o^*}, \quad N_{S,P} = P_* = \frac{\chi_*^2 (f Re)_*}{\varepsilon_*^2}$$

The general performance criterion  $N_S P_*$  become:

$$N_S P_* = \frac{1}{1 + \phi_o} \frac{\chi_*^2 (f Re)_*}{\varepsilon_*^2} \left( \frac{\chi_*^{1/2} g_o^*}{T_o^*} + \phi_o P_* \right) \quad (22)$$

The calculated values of  $P_*$  by Eq. (20) are shown in Fig. 3(a), and as seen  $P_* > 1$  for all cases.

Consequently, for case FG-3 it is not possible to decrease the pumping power of the unit using non-circular channels investigated in this study.

Figs. 3(b)-3(c) present the variation of  $N_S P_*$  with  $\chi_*$  and  $\phi_o$  as a parameter. As seen, the values of  $N_S P_*$  are greater than unity for all  $\chi_*$  and  $\phi_o$  considered. There is only one exception, rectangular channels with  $\chi_* > 1.65$  and  $\phi_o = 10^{-3}$ .

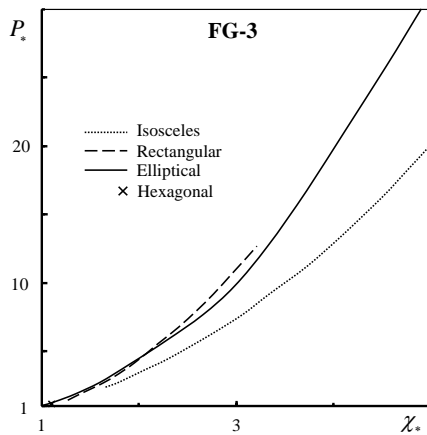


Fig. 3(a). The variation of  $P_*$  with  $\chi_*$

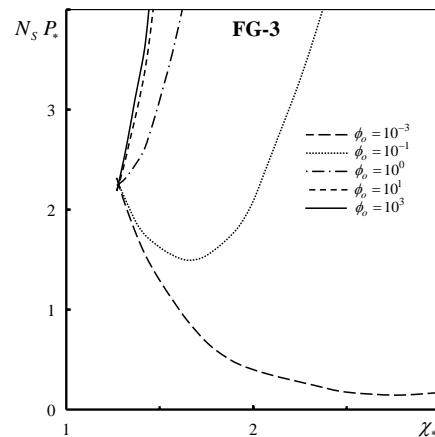


Fig. 3(b). The variation of  $N_S P_*$  with  $\chi_*$  (rectangular ducts)

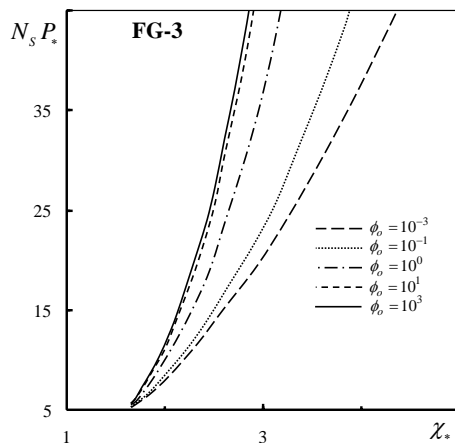


Fig. 3(c). The variation of  $N_S P_*$  with  $\chi_*$  (isosceles triangular ducts)

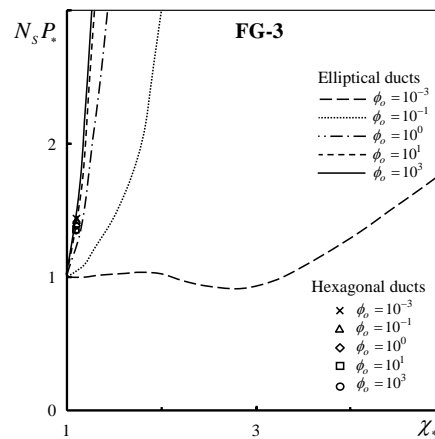


Fig. 3(d). The variation of  $N_S P_*$  with  $\chi_*$  (elliptical and hexagonal ducts)

Consequently, in this case the circular tube configuration is the best choice for heat exchanger.

### III.2. Fixed Number of Tubes Criteria (FN)

These criteria maintain constant number of tubes. The objective of FN-1 is reduced surface area, via reduced tubing length, for constant pumping power. Reduced flow rate will probably be required to satisfy the constant pumping power criterion. The objective of FN-2 is reduced pumping power.

#### III.2.1. Case FN-1

The objective functions of the case FN-1 are reduced heat transfer area  $A_* < 1$  ( $L_* < 1$ ), decreased entropy generation number  $N_S < 1$ , and simultaneous effect of the both of them  $N_S A_* < 1$ .

The constraints imposed are:  $N_* = 1$ ,  $Q_* = 1$ ,  $P_* = 1$ ,  $\Delta T_i^* = 1$ , and  $A_f = 1$ .

The consequences of the constraints are  $D_* < 1$ ,  $Re_* < 1$ . The relative Eqs. (2)-(6) yield:

$$W_* = \varepsilon_*^{-1} \quad (23)$$

$$\varepsilon_* = 1.0265 \left[ 1 - \exp \left( -Nu \frac{\varepsilon_*^2}{(f Re)_*} \right) \right] \quad (24)$$

$$L_* = \frac{\varepsilon_*^2}{\chi_* (f Re)_*} \quad (25)$$

$$A_* = \frac{\varepsilon_*^2}{\chi_*^{1/2} (f Re)_*} \quad (26)$$

Eq. (24) is to be solved by an iterative procedure for any  $Nu$ :

$$T_o^* = 0.981 + 0.019\varepsilon_*$$

$$\mathcal{G}_o^* = \exp \left\{ 3.657 \left[ 1 - Nu_* \frac{\varepsilon_*^3}{(f Re)_*} \right] \right\} \quad (27)$$

$$N_{S,T} = \mathcal{G}_o^* T_o^{*-1}, \quad N_{S,P} = P_* = 1$$

The general performance criterion  $N_S A_*$  becomes:

$$N_S A_* = \frac{1}{1 + \phi_o} \frac{\varepsilon_*^2}{\chi_*^{1/2} (f Re)_*} \left( \frac{\mathcal{G}_o^*}{T_o^*} + \phi_o \right) \quad (28)$$

The calculated values of  $A_*$  by Eqs. (26) and presented in Fig. 4(a) are in the ranges: for rectangular ducts  $A_* = 0.965 - 0.445$ ; for isosceles triangular ducts -  $A_* = 0.809 - 0.717$ ; for elliptical ducts -  $A_* = 1.0 - 0.290$ ; for hexagonal ducts -  $A_* = 0.999$ .

While investigating the behavior of isosceles triangular ducts an unexpected phenomenon arose. For  $\chi_* > 1.85$  the constraint  $P_* = 1$  cannot be maintained and the solutions of Eq. (24) do not have physical meaning. Consequently, it is possible to decrease the heat transfer area  $A_* < 1$  only of the unit using rectangular or elliptical ducts. Figs. 4(b)-4(d) present the variation of  $N_S A_*$  with  $\chi_*$  and  $\phi_o$  as a parameter. As seen, for heat exchanger with rectangular ducts, Fig. 4(b), for  $\chi_* \approx 1.7$  the behavior of the unit does not depend on the value of  $\phi_o$ .

For  $\chi_* < 1.7$ , some small benefit can be achieved only for  $\phi_o \gg 1$ . If the ducts are with  $\chi_* > 1.7$ ,  $N_S A_* < 1$  for all values of  $\chi_*$  and  $\phi_o$ .

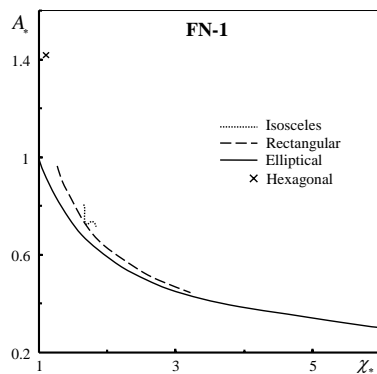


Fig. 4(a). The variation of  $A_*$  with  $\chi_*$ .

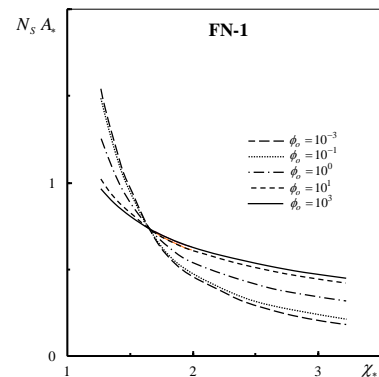


Fig. 4(b). The variation of  $N_S A_*$  with  $\chi_*$  (rectangular ducts)

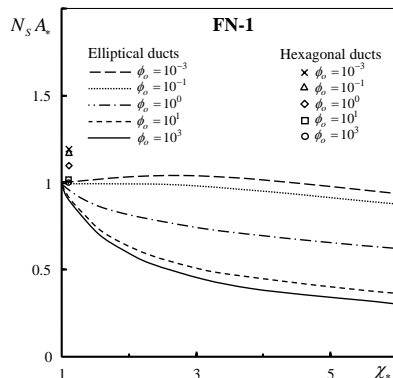


Fig. 4(c). The variation of  $N_S A_*$  with  $\chi_*$  (elliptical and hexagonal ducts)

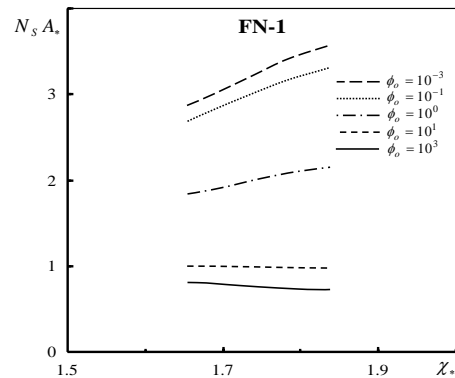


Fig. 4(d). The variation of  $N_S A_*$  with  $\chi_*$  (isosceles triangular ducts)



The benefit increases with the increase of  $\chi_*$  and decrease of  $\phi_o$ , and the best choice is the duct with  $\chi_* = 3.223$ .

Fig. 4(c) shows the results for elliptical and hexagonal ducts. As seen, the variation of  $N_S A_*$  with  $\phi_o$  for elliptical ducts is opposite of that of the rectangular ducts, and the best performance can be obtained by ducts with  $\chi_* = 6.274$  and  $\phi_o > 1$ .

The use of hexagonal ducts or isosceles triangular ducts, Fig. 4(c), instead of circular tubes is not able to bring any improvement of the efficiency of the heat exchanger.

### III.2.2. Case FN-2

The objective functions of the case FN-2 are lower pumping power  $P_* < 1$ , decreased entropy generation number  $N_S < 1$ , and simultaneous effect of the both of them  $N_S P_* < 1$ .

The constraints imposed are:  $Q_* = 1$ ,  $A_* = 1$ ,  $\Delta T_i^* = 1$ ,  $A_f^* = 1$ ,  $N_* = 1$ . The consequences of the constraints are  $D_* < 1$ ,  $L_* < 1$ . The relative Eqs. (2)-(6) yield  $L_* = \chi_*^{-1}$ ,  $Re_* = \chi_*^{-1/2} \varepsilon_*^{-1}$ ,  $W_* = \varepsilon_*^{-1}$ :

$$\varepsilon_* = 1.0265 \left[ 1 - \exp(-Nu \chi_*^{1/2} \varepsilon_*) \right] \quad (29)$$

$$P_* = \frac{\chi_*^{1/2} (f Re)_*}{\varepsilon_*} \quad (30)$$

$$g_o^* = \exp \left[ 3.657 \left( 1 - Nu_* \chi_*^{1/2} \varepsilon_* \right) \right] \quad (31)$$

$$T_o^* = 0.981 + 0.019 \varepsilon_*; N_{S,T} = g_o^* T_o^{*-1}$$

$$N_{S,P} = P_* = \frac{\chi_*^{1/2} (f Re)_*}{\varepsilon_*}$$

Eq. (29) is to be solved by iterative process for any  $Nu$ . The general performance criterion  $N_S P_*$  becomes:

$$N_S P_* = \frac{1}{1 + \phi_o} \frac{\chi_*^{1/2} (f Re)_*}{\varepsilon_*} \left[ \frac{g_o^*}{T_o^*} + \phi_o \frac{\chi_*^{1/2} (f Re)_*}{\varepsilon_*} \right] \quad (32)$$

The calculated values of  $P_*$  by Eq. (30), and shown in Fig. 5(a), revealed that in all cases  $P_* > 1$  and the first objective  $P_* < 1$  cannot be achieved.

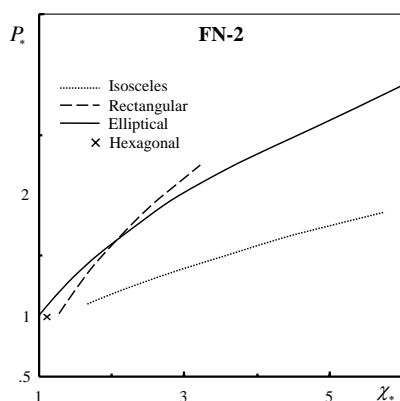


Fig. 5(a). The variation of  $P_*$  with  $\chi_*$

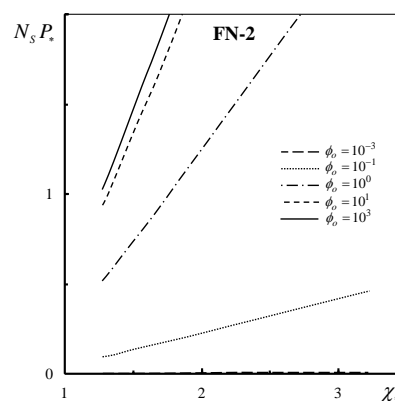


Fig. 5(b). The variation of  $N_S P_*$  with  $\chi_*$  (rectangular ducts)

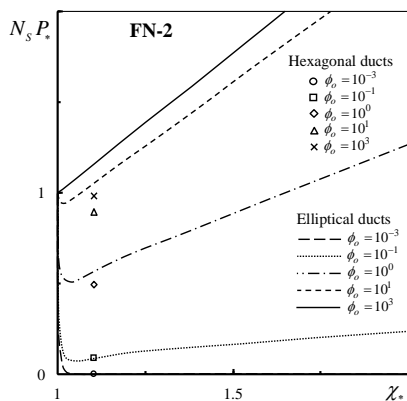


Fig. 5(c). The variation of  $N_S P_*$  with  $\chi_*$  (elliptical and hexagonal ducts)

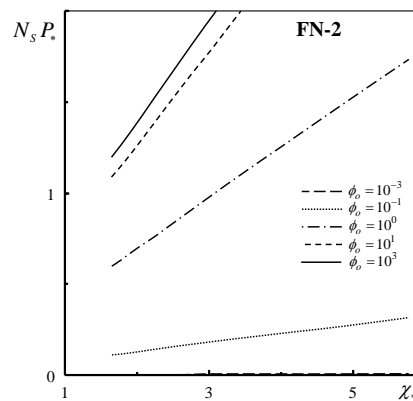


Fig. 5(d). The variation of  $N_S P_*$  with  $\chi_*$  (isosceles triangular ducts)

The use of the general criterion, Figs. 5(b)-5(d), however, has shown that benefits can be obtained even though with isosceles triangular ducts for small values of  $\phi_o$  ( $\phi_o < 1$ ) and the benefit increases with the decrease of  $\chi_*$ . In this case, the use of elliptical ducts looks as the most appropriate.

### III.3. Variable Geometry Criteria (VG)

The criteria VG are applicable when the heat exchanger is “sized” for a required thermal duty with specified flow rate.

#### III.3.1. Case VG-1

The objective functions of the case VG-1 are lower heat transfer area  $A_* < 1$ , decreased entropy generation number  $N_S < 1$ , and simultaneous effect of the both of them  $N_S A_* < 1$ .

The constraints imposed are:  $Q_* = 1$ ,  $W_* = 1$ ,  $P_* = 1$ ,  $\Delta T_i^* = 1$ ,  $A_f^* = 1$ . The consequences of the constraints are  $L_* < 1$ ,  $D_* < 1$ ,  $V_* < 1$ . The relative Eqs. (2)-(6) yield:  $\varepsilon_* = 1$ ,  $T_o^* = 1$ :

$$g_o^* = \exp\left[3.657\left(1 - \chi_*^{1/2}\right)\right] \quad (33)$$

$$A_* = Nu_*^{-1} \quad (34)$$

(since  $Q_* = 1$ , and  $\Delta T_m^* = 1$ ):

$$N_* = \left[\frac{\chi_*^{1/2} (f Re)_*}{Nu_*}\right]^{1/3} \quad (35)$$

$$L_* = \left[\chi_*^2 (f Re)_* Nu_*^2\right]^{-1/3} \quad (36)$$

$$Re_* = \left[\frac{Nu_*}{\chi_* (f Re)_*}\right]^{1/3} \quad (37)$$

$$N_{S,T} = \frac{g_o^*}{N_*}, N_{S,P} = 1$$

The general performance criterion  $N_S A_*$  becomes:

$$N_S A_* = \frac{1}{(1 + \phi_o) Nu_*} \left\{ \frac{g_o^*}{N_*} + \phi_o \right\} \quad (38)$$

The calculated values of  $A_*$  by Eq. (34) are presented in Fig. 6(a).

As seen, the use of rectangular ducts can only be profitable if  $\chi_* > 1.7$ , and the values of  $A_*$  decrease with the increase of  $\chi_*$ ,  $A_* = 0.654$  for  $\chi_* = 3.223$ . All other cases cannot decrease the heat transfer area of the unit. Nevertheless, the calculations made by use of the general criterion revealed that the objective  $N_S A_* < 1$  can be achieved and it depends on the values of  $\chi_*$  and  $\phi_o$ . For rectangular ducts (Fig. 6(b)) if  $\phi_o \leq 1$ ,  $N_S A_* < 1$  for all values of  $\chi_*$ , and the benefit increases with the increase of  $\chi_*$ . If  $\phi_o > 1$ , profit can be obtained for  $\chi_* > 1.7$  and it increases with the increase of  $\chi_*$ . For elliptical ducts (Fig. 6(c)), a small benefit can be obtained even though  $\phi_o > 1$ , but substantial improvement can be reached if  $\phi_o \ll 1$  and  $\chi_* > 1.2$ .

The unit with elliptical ducts, Fig. 6(c), performs almost in the same way as that one with rectangular ducts but not better. The use of isosceles ducts, Fig. 6(d), can also be profitable if  $\phi_o < 1$  and the benefit increases with the increase of  $\chi_*$  and decrease of  $\phi_o$ .

#### III.3.2. Case VG-2a

The objective functions of the case VG-2a are increased heat duty  $Q_* > 1$ , decreased entropy generation numbers  $N_S < 1$ , and simultaneous effect of the both of them  $N_S / Q_* < 1$ .

The constraints imposed are:  $W_* = 1$ ,  $\Delta T_i^* = 1$ ,  $P_* = 1$ ,  $A_* = 1$ ,  $A_f^* = 1$ . The consequences of the constraints are  $D_* = 1$ ,  $N_* > 1$ ,  $L_* < 1$ ,  $V_* < 1$ .

The relative Eqs. (2)-(6) yield:

$$L_* = \left[\chi_*^2 (f Re)_*\right]^{-1/3} \quad (39)$$

$$N_* = \left[\chi_*^{1/2} (f Re)_*\right]^{1/3} \quad (40)$$

$$Q_* = \varepsilon_* \quad (41)$$

$$\varepsilon_* = 1.0265 \left[1 - \exp\left(-Nu \chi_*^{1/2}\right)\right]$$

$$g_o^* = \exp\left[3.657\left(1 - \chi_*^{1/2} Nu_*\right)\right]$$

$$T_o^* = 0.981 + 0.019 \varepsilon_*$$

The general performance criterion  $N_S / Q_*$  becomes:

$$N_S / Q_* = \frac{1}{1 + \phi_o} \left\{ \frac{g_o^*}{N_* T_o^*} + \frac{\phi_o}{\varepsilon_*} \right\} \quad (42)$$

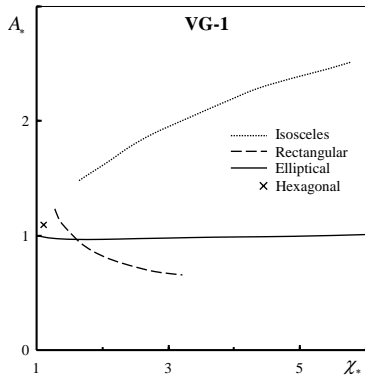


Fig. 6(a). The variation of  $A_s$  with  $\chi_*$

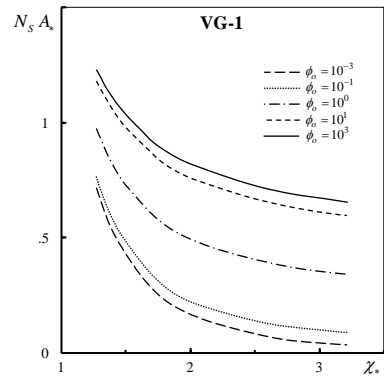


Fig. 6(b). The variation of  $N_s A_s$  with  $\chi_*$  (rectangular ducts)

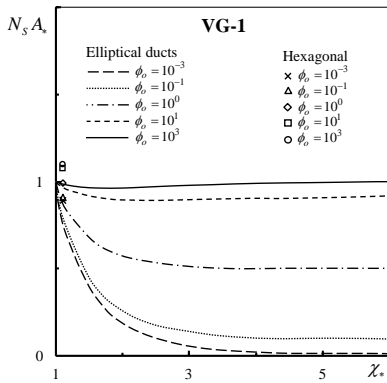


Fig. 6(c). The variation of  $N_s A_s$  with  $\chi_*$  (elliptical and hexagonal ducts)

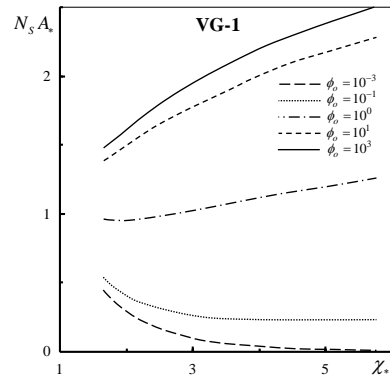


Fig. 6(d). The variation of  $N_s A_s$  with  $\chi_*$  (isosceles triangular ducts)

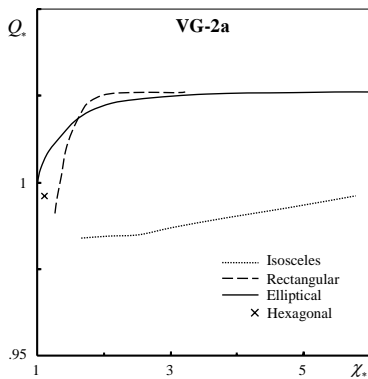


Fig. 7(a). The variation of  $Q_s$  with  $\chi_*$

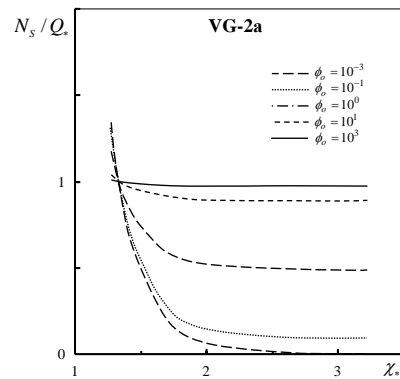


Fig. 7(b). The variation of  $N_s / Q_s$  with  $\chi_*$  (rectangular ducts)

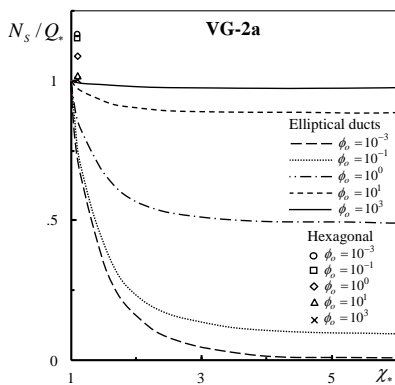


Fig. 7(c). The variation of  $N_s / Q_s$  with  $\chi_*$  (elliptical and hexagonal ducts)

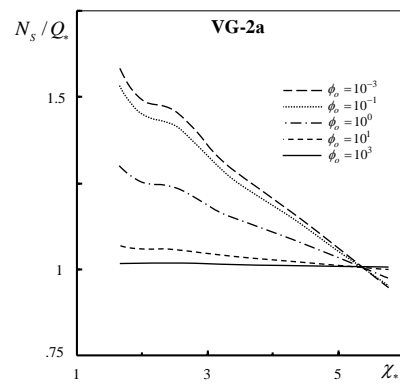


Fig. 7(d). The variation of  $N_s / Q_s$  with  $\chi_*$  (isosceles triangular ducts)

The values of  $Q_*$ , calculated by Eq. (41) are shown in Fig. 7(a). As seen, the first objective  $Q_* > 1$  can only be achieved by elliptical and rectangular (if  $\chi_* > 1.7$ ) ducts and small benefit can be obtained.

The use of isosceles or hexagonal ducts is completely inappropriate for this goal.

Figs. 7(b)-7(c) present the variation of  $N_S/Q_*$  with  $\chi_*$  and  $\phi_o$  as a parameter. As seen, for heat exchanger with rectangular ducts, Fig. 7(b), the objective  $N_S/Q_* < 1$  can be achieved for  $\chi_* > 1.45$ , and the benefit increases with the increase of  $\chi_*$  and decrease of  $\phi_o$ . For ducts with  $\chi_* \approx 1.35$  the value of  $N_S/Q_*$  almost does not depend on the value of  $\phi_o$ . Heat exchanger with elliptical ducts, Fig. 7(c), will perform almost in the same way as unit with rectangular ducts and better than heat exchanger with circular tubes despite of the shape ratio  $\chi_*$  and the value of  $\phi_o$ . For isosceles triangular ducts, Fig. 7(d), and hexagonal ducts, Fig. 7(c), the objective  $N_S/Q_* < 1$  cannot be achieved.

#### IV. Conclusion

Extended performance evaluation criteria (ExPEC) have been used to assess the benefits of the use of different non-circular ducts instead of circular tubes in heat exchangers with single-phase fully developed laminar flow. The performance characteristics of rectangular, isosceles triangular, elliptical and hexagonal ducts have been evaluated for different objectives and constraints imposed. The constant wall temperature has been selected as thermal boundary condition. As a common constraint, the cross sectional area of the non-circular duct has been specified,  $A_f^* = 1$ . The results can be summarized as follows:

1. The performance characteristics of the unit with non-circular ducts strongly depend on the objectives pursued, constraints imposed, shape factor  $\chi_*$ , and irreversibility distribution ratio  $\phi_o$ . In this regard, in many of the cases studied the use of rectangular or elliptical ducts are able to be more profitable than the use of circular tubes.
2. In the cases FG, the use of rectangular or elliptical ducts could be profitable only in FG-1a if  $\phi_o \ll 1$ , i.e., the entropy generated by final temperature difference is much greater than that of hydraulic friction. In this case, the heat exchanger with rectangular channels and aspect ratio of  $a/b = 16$  realizes the greatest efficiency.
3. In the case FN-1, the use of rectangular or elliptical ducts could also be beneficial. If  $\phi_o < 1$ , rectangular ducts with  $\chi_* > 1.7$  could be used and the benefit will increase with the increase of  $\chi_*$  and decrease of  $\phi_o$ . If  $\phi_o > 1$ , elliptical ducts are more profitable the benefit increases with the increase of  $\chi_*$  and  $\phi_o$ .
4. In the case VG-1, the use of rectangular ducts is definitely better choice. For  $\chi_* > 1.7$ ,  $A_* < 1$  and profit can be obtained even though  $\phi_o > 1$ . If  $\phi_o \leq 1$ ,  $N_S A_* < 1$  for all values of  $\chi_*$ , and the benefit increases with the increase of  $\chi_*$ .
5. In the case VG-2a, the rectangular and elliptical ducts have better performance characteristics than circular tubes. The analysis of the performances of heat exchangers with non-circular channels revealed that the use of rectangular or elliptical ducts could compete, in some case, the circular tube configuration for single-phase laminar fully developed flow through the tubes. The choice of the duct shape depends on the constraints imposed and the objectives pursued. The results obtained from the present study have confirmed again the importance of the use of ExPEC to assess the benefits and select the optimal heat exchanger design.

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